Solution 1.

(a) **Base case:** \((n = 1)\): The value of the summation is

\[
\sum_{i=1}^{1} i(i + 1) = 1(1 + 1) = 2 .
\]

The formula gives

\[
\frac{1(1+1)(1+2)}{3} = 2 .
\]

Since these are equal we have proved the base case.

**Induction step:** Assume the formula holds for \(n - 1\). This implies that

\[
\sum_{i=1}^{n-1} i(i + 1) = \frac{(n - 1)n(n + 1)}{3} .
\]

Thus,

\[
\sum_{i=1}^{n} i(i + 1) = \sum_{i=1}^{n-1} i(i + 1) + n(n + 1) = \left( \frac{(n - 1)n(n + 1)}{3} \right) + n(n + 1) \quad \text{by the induction hypothesis}
\]

\[
= \frac{(n - 1)n(n + 1)}{3} + \frac{3n(n + 1)}{3}
\]

\[
= \frac{(n + 2)n(n + 1)}{3} = \frac{n(n + 1)(n + 2)}{3} .
\]

This is the desired formula.

(b) **Base case:** \((n = 0)\): The value of the summation is

\[
\sum_{i=0}^{0} 2^i = 2^0 = 1 .
\]

The formula gives

\[
2^{0+1} - 1 = 2 - 1 = 1 .
\]

**Induction step:** Assume the formula holds for \(n - 1\). This implies that \(\sum_{i=0}^{n-1} 2^i = 2^n - 1\). Thus,

\[
\sum_{i=0}^{n} 2^i = \left( \sum_{i=0}^{n-1} 2^i \right) + 2^n = (2^n - 1) + 2^n \quad \text{by the induction hypothesis}
\]

\[
= 2 \cdot 2^n - 1 = 2^{n+1} - 1
\]
Solution 2.

(a) \( x = \log_b a \).

(b) By part (a), \( a = c^{\log_c a} \), \( b = c^{\log_c b} \), and \( ab = c^{\log_c(ab)} \). So,
\[
    c^{\log_c(ab)} = ab = c^{\log_c a} \cdot c^{\log_c b} = c^{\log_c a + \log_c b}.
\]
Since, exponentiation to the same base \( c \neq 1 \) is one-to-one, \( \log_c(ab) = \log_c a + \log_c b \).

(c) Since the log function is one-to-one, to show that \( f(n) = g(n) \), it suffices to show that \( \log_b f(n) = \log_b g(n) \):
\[
    \log_b a^{\log_b n} = (\log_b n)(\log_b a) = (\log_b a)(\log_b n) = \log_b (n^{\log_b a})
\]

Solution 3.

(a)
\[
    \frac{2x}{x^2 + 5}
\]

(b) \( \lg(x^2 + 5) = \frac{\ln(x^2+5)}{\ln 2} \), so the derivative is
\[
    \frac{2x}{(x^2 + 5) \ln 2} = \frac{2x \ln e}{x^2 + 5}
\]

(c) \( \frac{1}{\ln(x^2+5)} = (\ln(x^2 + 5))^{-1} \), so the derivative is
\[
    - (\ln(x^2 + 5))^{-2} \cdot \frac{1}{x^2 + 5} \cdot 2x = -\frac{2x}{(x^2 + 5)(\ln(x^2 + 5))^2}
\]
Solution 4. (a)

\[ \int \frac{1}{x} \, dx = \ln x + C \]

(b) Use substitution. Let \( y = 3x + 7 \). Then \( dy = 3 \, dx \). So,

\[ \int \frac{1}{3x + 7} \, dx = \frac{1}{3} \int \frac{dy}{y} = \frac{\ln y}{3} + C = \frac{\ln(3x + 7)}{3} + C \]

(c) Let \( u = \ln x \) and \( dv = dx \). Then, \( du = dx/x \) and \( v = x \). So,

\[ \int \ln x \, dx = x \ln x - \int \frac{x \, dx}{x} = x \ln x - \int dx = x \ln x - x + C \]

(d) Let \( u = \ln x \) and \( dv = x \, dx \). Then, \( du = dx/x \) and \( v = x^2/2 \). So,

\[ \int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \frac{dx}{x} = \frac{x^2 \ln x}{2} - \int \frac{x}{2} \, dx = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C \]

(e) By part (d) and the fact that \( x \lg x = \frac{x \ln x}{\ln 2} \),

\[ \int x \lg x \, dx = \frac{1}{\ln 2} \int x \ln x \, dx = \frac{x^2 \ln x}{2 \ln 2} - \frac{x^2}{4 \ln 2} + C = \frac{x^2 \lg x}{2} - \frac{x^2 \lg e}{4} + C \]