Questions?

• Final
  – In class final on May 18th from 10:30 to 12:30

• Preliminary findings presentation
  – 04/24

• Paper review in class
  – Be ready to have a draft by May 1st

• Final presentation
  – 05/13
One-way Independent ANOVA

• Setting
  – $k$ different treatments: (pen versus mouse, versus TrackPoint)
  – For each treatment you gathered from a random samples
    • Mean
    • Standard deviation

• Null hypothesis
  – The $F$ ratio follows a $F$ distribution

\[
F = \frac{MS_{bg}}{MS_{wg}}, \quad \text{with} \quad \begin{cases} 
MS_{bg} = \frac{SS_{bg}}{df_{bg}}, & SS_{bg} = \sum_{j=1}^{k} n_j (\overline{X}_j - \overline{X}_G)^2 \\
MS_{wg} = \frac{SS_{wg}}{df_{wg}}, & SS_{wg} = \sum (n_j)\sigma_j^2 \\
df_{bg} = k - 1 \\
df_{wg} = N - k
\end{cases}
\]

From Explaining Psychological Statistic (Cohen)
Interpretation

\[ F = \frac{\text{treatment effect} + \text{error variance}}{\text{error variance}} \]

From Explaining Psychological Statistic (Cohen)
$F$ distribution

From Explaining Psychological Statistic (Cohen)
Two-way ANOVA

• Setting
  – Full factorial design: \( k \) columns, \( l \) rows
  – For each cell (treatment) you gathered from a random samples
    • Mean
    • Standard deviation

• Null hypothesis
  – \( F \) ratios (row column and interaction) follow a \( F \) distribution
    • Main effect: \( F_{\text{col}} \) and \( F_{\text{row}} \)
    • Interaction: \( F_{\text{int}} \)
F ratios

\[ F_{\text{col}} = \frac{MS_{\text{col}}}{MS_{\text{cell}}} \]

with

\[
\begin{align*}
MS_{\text{col}} &= \frac{SS_{\text{col}}}{df_{\text{col}}} \\
SS_{\text{col}} &= n_{\text{col}} \sum_{j=1}^{k} (X_j - \bar{X}_G)^2 \\
df_{\text{col}} &= \text{col\_count} - 1
\end{align*}
\]

\[ F_{\text{row}} = \frac{MS_{\text{row}}}{MS_{\text{cell}}} \]

with

\[
\begin{align*}
MS_{\text{row}} &= \frac{SS_{\text{row}}}{df_{\text{row}}} \\
SS_{\text{row}} &= n_{\text{row}} \sum_{j=1}^{k} (X_j - \bar{X}_G)^2 \\
df_{\text{col}} &= \text{row\_count} - 1
\end{align*}
\]

\[ F_{\text{int}} = \frac{MS_{\text{int}}}{MS_{\text{cell}}} \]

with

\[
\begin{align*}
MS_{\text{int}} &= \frac{SS_{\text{int}}}{df_{\text{int}}} \\
SS_{\text{int}} &= \left( \sum_{j=1}^{k} \frac{T_j^2}{n_j} - \frac{T^2}{N} \right) - SS_{\text{row}} - SS_{\text{col}} \\
df_{\text{int}} &= df_{\text{row}} \times df_{\text{col}}
\end{align*}
\]

and

\[
\begin{align*}
MS_{\text{cell}} &= \frac{SS_{\text{cell}}}{\text{cell\_count}} \\
SS_{\text{cell}} &= \sum \sigma_j^2, \text{ assuming equal size cells} \\
df_{\text{cell}} &= N_T - \text{cell\_count}
\end{align*}
\]
Interpretation (General Linear Model)

Score = Grand mean 
+ Row effect 
+ Column effect 
+ Interaction effect 
+ Error
Two-way ANOVA example
Interactions

From Explaining Psychological Statistic (Cohen)
Interactions (Results)

$C =$ Column means
$R =$ Row means

Interaction is not significant

Interaction is significant
Interactions (Caveats)

- If the interaction is significant
  - Main effect must be interpreted with care
- If the interaction is not significant
  - Main effect can be interpreted with 2 separate ANOVAs
Other tests

• Repeated measure ANOVA
  – For within subject design

• Mixed Design ANOVA
  – For design which are both between and within subjects

• Multiple comparisons
  – Tukey and Bonferroni correction

• Nonparametric Statistics
  – Distribution-free tests
    • *non-ordinal scale*
    • *nominal scale*
    • *non-normal distribution*
Paper discussion