1. Use the following propositions for this problem.
   • $b = \text{"John is a basketball player."}$
   • $h = \text{"John is a hard worker."}$
   • $c = \text{"John is a computer science major."}$

Write each of the following statements in symbolic form using the symbols $\sim$, $\land$, and $\lor$ and the letters defined above.

(a) John is a basketball player, however he is not a computer science major.
   \textbf{Answer: } $b \land \sim c$

(b) John is either a hard worker or a basketball player, but not both.
   \textbf{Answer: } $(h \land \sim b) \lor (\sim h \land b)$ OR $(h \lor b) \land \sim (h \land b)$

(c) John is a hard worker, but he is neither a basketball player nor a computer science major.
   \textbf{Answer: } $h \land \sim b \land \sim c$

2. Use DeMorgan’s Laws to write the negations of the following as English sentences.

(a) Mary and Sam are both engineering majors.
   \textbf{Answer: } Mary is not an engineering major or Sam is not an engineering major.

(b) Morgan is is neither a swimmer nor a runner.
   \textbf{Answer: } Morgan is either a swimmer or a runner (or both).

(c) Either the weather is nice or the game is cancelled.
   \textbf{Answer: } The weather is not nice and the game is not cancelled.

3. Assuming $x$ is a particular real number,

(a) write the negation of this statement: $-1 < x \leq 4$.
   \textbf{Answer: } $(-1 \geq x) \lor (x > 4)$

(b) Give one possible value that $x$ could be if the statement “$-1 < x \leq 4$” is false.
   \textbf{Answer: } Many possible answers. Any number less than or equal to -1 or greater than 4 is fine.

4. State whether each of the following sentences are statements. If a sentence is a statement, state whether it is true or false.

(a) “Why is the sky blue?”
   \textbf{Answer: } Not a statement — it’s a question.

(b) “My CMSC250 instructor is Jan Plane.”
   \textbf{Answer: } True statement.

(c) “This sentence is not a statement.”
Answer: False statement.

(d) “This sentence is both true and false.”
Answer: False statement.

5. Construct a complete a truth table for each of the following statements, and tell whether each is a tautology, contradiction, or neither.

(a) \((p \land \sim q) \land (\sim p \lor q)\)

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>\sim p</th>
<th>\sim q</th>
<th>p \land \sim q )(X)</th>
<th>\sim p \lor q )(Y)</th>
<th>(X) \land (Y)</th>
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Answer: 1 0 0 1 1 0 0

This statement is a contradiction.

(b) \(\sim p \land (q \lor \sim r)\)

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<th>q</th>
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<th>\sim p</th>
<th>q \lor \sim r</th>
<th>\sim p \land (q \lor \sim r)</th>
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Answer: 1 0 0 0 1 1 0

This statement is a neither a tautology nor a contradiction.

(c) \((p \land \sim q) \lor (p \land q)\lor \sim p\)

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<tr>
<th>p</th>
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<th>p \land q )(Y)</th>
<th>(X) \lor (Y)</th>
<th>(X) \lor (Y) \lor \sim p</th>
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This statement is a tautology.