Due Wednesday, March 17 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Simplify this expression: \( \ln \left( \prod_{i=1}^{n} e^{f(i)} \right) \).

2. Prove \( \forall n \in \mathbb{Z}^+ \sum_{i=1}^{n} (2i)(2i - 1) = \frac{n(n + 1)(4n - 1)}{3} \).

3. Prove \( \forall n \in \mathbb{Z}^+ \sum_{i=1}^{n} (4i - 3) = n(2n - 1) \).

   Note: Do not use Theorem 4.2.2 to solve this problem.

4. Prove \( \forall n \in \mathbb{Z}^+ \prod_{i=1}^{n} 2^i = 2 \left( \frac{n^2 + n^3}{2} \right) \).

5. Recall the recursive definition of the Fibonacci sequence:

   \[
   F_1 = 1 \\
   F_2 = 1 \\
   F_k = F_{k-1} + F_{k-2} \quad \text{for} \ k > 2.
   \]

   Prove these interesting facts about this sequence:

   (a) \( \forall n \in \mathbb{Z}^+ \sum_{k=1}^{n} F_k = F_{n+2} - 1 \)

   (b) \( \forall n \in \mathbb{Z}^+ \sum_{k=1}^{n} F_k^2 = F_n \cdot F_{n+1} \).

   Hint: You can perform a change of variable on the third line in the Fibonacci sequence formula to obtain equivalent formulas that may prove useful.