Due Wednesday, April 7 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

Note: $\mathcal{P}(A)$ denotes the power set of $A$.

1. For each of the following, give a proof of the statement if it is true, or a counterexample if the statement is false. Remember, counterexamples must include specific values and enough work shown to demonstrate that they are actual counterexamples.

   (a) For all sets $A$, $B$, and $C$, if $A \cup C = B \cup C$, then $A = B$.
   (b) For all sets $A$, $B$, and $C$, if $A \subseteq B$ and $B \subseteq C$, then $A \times B \subseteq B \times C$.
   (c) For all sets $A$, $B$, and $C$, $(A \setminus B) - (B \setminus C) = A - B$.
   (d) For all sets $A$ and $B$, if $A \cap B = \emptyset$ then $A \times B = \emptyset$.
   (e) For all sets $A$, $B$, and $C$, if $B \cap C \subseteq A$, then $(C \setminus A) \cap (B \setminus A) = \emptyset$.
   (f) For all sets $A$ and $B$, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

2. Let $A_1, A_2, \ldots$ be sets. Prove the generalized DeMorgan’s law:

   $$\forall n \in \mathbb{Z}^+ \ (A_1 \cup A_2 \cup \cdots \cup A_n)' = A'_1 \cap A'_2 \cap \cdots \cap A'_n$$

   Hint: Use induction, and the facts that

   - $A_1 \cup A_2 \cup \cdots \cup A_n = A_1 \cup A_2 \cup \cdots \cup A_{n-1} \cup A_n$.
   - $A_1 \cap A_2 \cap \cdots \cap A_n = A_1 \cap A_2 \cap \cdots \cap A_{n-1} \cap A_n$.

3. Let $B_1, B_2, \ldots$ be sets.

   Let $B_1 = \{0, 1\}$, $B_2 = \{1, 2\}$, and $\forall i \in \mathbb{Z}^2 \ B_i = (B_{i-1} \cup \{i\}) - B_{i-2}$.

   Prove $\forall n \in \mathbb{Z}^+ \ B_n = \{n-1, n\}$.