Password for today: minnow
Let \( a_0 = 2, \ a_1 = 7 \). For \( k \geq 2 \), let \( a_k = 12a_{k-1} + 3a_{k-2} \)

We want to find integers \( A, B \) such that: For all \( n \geq 0 \): \( a_n \leq AB^n \)

Sketch of Proof: I will apply strong induction on \( n \).

Base Case (\( n = 0, 1 \)): \( a_0 = 2 \leq AB^0 \), \( 2 \leq A \)

\( a_1 = 7 \leq AB^1 \), \( 7 \leq AB \)

Inductive Hypothesis: Let \( k \geq 1 \). Assume (for all \( i \leq k \)) that \( a_i \leq AB^i \)

Inductive Step: [I must show \( a_{k+1} \leq AB^{k+1} \)]

\( a_{k+1} = [\text{From defn of sequence}] \)

\[ 12a_k + 3a_{k-1} \leq [\text{By I.H.}] \]

\[ 12AB^k + 3AB^{k-1}. \]

So we need: \( 12AB^k + 3AB^{k-1} \leq AB^{k+1} \)

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So we need: \(12AB^k + 3AB^{k-1} \leq AB^{k+1}\)

Divide through by \(AB^{k-1}\): \(12B + 3 \leq B^2\)

\(0 \leq B^2 - 12B - 3\)

What (integer) values of \(B\) make this true? (Graph it?)

Now we know:

\(B \geq 13\)

\(A \geq 2\)

\(AB \geq 7\)

Conclusion?

We can show by induction that \((\forall n \in \mathbb{Z}^{\geq 0})[a_n \leq 2 \cdot 13^n]\)