Due at the start of class Friday, June 6, 2003.

**Problem 1.** Use mathematical induction to show that

\[ (a) \quad \sum_{i=1}^{n} i(i + 1) = \frac{n(n + 1)(n + 2)}{3} \]

\[ (b) \quad \sum_{i=0}^{n} 2^i = 2^{n+1} - 1 \]

**Problem 2.** See bottom of page 53 of CLRS (bottom of page 34 in CLR) and/or other side of this sheet.

(a) Assume \( b^x = a \). What is \( x \) (in terms of \( a \) and \( b \))?

(b) Using only part (a), show that \( \log_c(ab) = \log_c a + \log_c b \).

(c) Show that \( a^{\log_b n} = n^{\log_b a} \)

**Problem 3.** Differentiate the following functions:

(a) \( \ln(x^2 + 5) \)

(b) \( \lg(x^2 + 5) \)

(c) \( \frac{1}{\ln(x^2 + 5)} \)

**Problem 4.** Integrate the following functions:

(a) \( \frac{1}{x} \)

(b) \( \frac{1}{3x+7} \)

(c) \( \ln x \) [HINT: Use integration by parts.]

(d) \( x \ln x \) [HINT: Use integration by parts.]

(e) \( x \lg x \)

**Problem 5.** Consider the problem of not only finding the value of the maximum contiguous sum in an array, but also determining the two endpoints. Give a linear time algorithm for solving this problem. [What happens if all entries are negative?]
\[
\begin{align*}
\lg n &= \log_2 n \\
\ln n &= \log_e n \\
\lg^k n &= (\lg n)^k \\
\lg \lg n &= \lg(\lg n)
\end{align*}
\]

For all real \(a > 0, b > 0, c > 0,\) and \(n,\)

\[
\begin{align*}
a &= b^{\log_b a} \\
\log_c(ab) &= \log_c a + \log_c b \\
\log_b a^n &= n \log_b a \\
\log_b a &= \frac{\log_c a}{\log_c b} \\
\log_b(1/a) &= -\log_b a \\
\log_b a &= \frac{1}{\log_a b} \\
a^{\log_b n} &= n^{\log_a a}
\end{align*}
\]