CMSC 351: Practice Questions for Midterm Exam

These are practice problems for the upcoming midterm exam. You will be given a sheet of notes for the exam. Also, go over your homework assignments. **Warning:** This does not necessarily reflect the length, difficulty, or coverage of the actual exam.

**Problem 1.** Do Problems 3.1-2 and 3.1-4 on page 50 of CLRS. (Problems 2.1-2 and 2.1-4 on page 31 of CLR.)

**Problem 2.** Show that \( n^2 - 7n + 1 = \Theta(5n^2 + 6n - 10) \) using the definition of \( \Theta \) given in class (do not use limits).

**Problem 3.** Use the iteration method to solve the following recurrences. You may assume \( n \) is “nice”. Prove your answers using mathematical induction.

(a) \( T(n) = 2T(n/2) + n^3, \quad T(1) = 1. \)

(b) \( T(n) = T(\sqrt{n}) + 1, \quad T(2) = 1. \)

(c) \( T(n) = 2T(n/2) + n \log n, \quad T(1) = 1. \)

(d) \( T(n) = T(n-3) + 5, \quad T(1) = 2. \)

**Problem 4.** Which of the above problems can be solved using the “Master Theorem” derived in class. Solve them exactly using the “Master Theorem”.

**Problem 5.** Show that

(a) \[
\frac{1}{2} \leq \sum_{j=1}^{\infty} \frac{1}{j2^j} \leq 1
\]

(b) \[
1 \leq \sum_{j=1}^{\infty} \frac{1}{j^2} \leq 2
\]

**Problem 6.** Consider a tree where every node has exactly zero or two children.

(a) How does the number of internal nodes (nonleaves) compare with the number of leaves?

(b) Prove your answer using mathematical induction.

**Problem 7.** How many times on average must you flip six fair coins before you obtain exactly three heads and three tails?
Problem 8. Let $a$ be a number such that $0 \leq a \leq 1$. Assume that you have a subroutine that finds the maximum of $n$ numbers in $(\log n)^a$ steps. Using this subroutine, write an algorithm that sorts a list of $n$ numbers. How fast does your algorithm run (you may use $\theta$-notation). For what values of $a$ is the algorithm better than $\theta(n \log n)$ (you may assume $n$ is large and reason asymptotically).

Problem 9. Do Problem 6-2 on page 143 of CLRS. (Problem 7-2 on page 152 of CLR.)