Problem 1:  Max-flow with forward edges only. From the book: Exercise 7.11.


Problem 4:  In the Degree $\Delta$ Spanning Tree Problem we are given a graph $G = (V, E)$. We are asked if there is a spanning tree of $G$ with degree at most $\Delta$. Note that $\Delta$ is not part of the input, but rather part of the problem definition. Prove that Degree 2 Spanning Tree is NP-complete. Prove that Degree $\Delta$ Spanning Tree is NP-complete for all $\Delta$.

Problem 5:  Cycle cover. Exercice 8.41.

Problem (extra credit):  Consider the following Network Design Problem. Let $G = (V, E)$ be an undirected graph representing a computer network. We would like to set up a virtual private network (VPN) for a subset of the computers $X \subset V$. If the graph induced$^1$ by $X$ is connected there is no problem setting up the VPN. However, if the graph is not connected additional intermediate routers must be used so that computers in $X$ can communicate with each other. These additional routes can be rented by paying a fixed fee. Since the fee is rather steep we would like to minimize their number of intermediate routers.

Here is a cleaner formulation of the problem we want to solve. Given a graph $G = (V, E)$, a set of terminals $X$ and a number $k$, we want to know if there is a set $S \subseteq V$ of size at most $k$ such that $G[X \cup S]$ is connected. Prove that this problem is NP-complete.

$^1$The graph induced by $X$ is defined as $G[X] = (X, E[X])$, where $E[X] = \{(u, v) \in E \mid u, v \in X\}$.