A Game-Theoretic Approach for Minimizing Delays in Autonomous Intersections

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Abstract Traffic management systems of the near future will be able to exploit communication between vehicles and autonomous traffic control systems to significantly improve the utilization of road networks. In this work, a novel game-theoretic model for the traffic management of vehicles in intersections is introduced. A core concept from game theory that captures the important interplay between independent decision making and centralized control is the notion of a correlated equilibrium. We characterize the correlated equilibria under this model, yielding interesting connections to maximum-weight independent sets in graphs. We develop efficient algorithms for computing optimal correlated equilibria and demonstrate through simulations the effectiveness of our algorithms for improving traffic flow.

1 Introduction

In this paper, we describe a model for intersection management using game-theoretic principles. The core of our model rests on the idea of a *correlated equilibrium* (CE) (see Section 2 for a formal definition). Here, the actions of agents are entrusted to an external entity, whose decisions – which may be probabilistic – satisfy the property that it is not in the interest of any agent to unilaterally deviate from the recommendations of this entity. In the context of traffic, drivers entrust their decisions to the traffic signals. It therefore makes intuitive sense that an intelligent traffic manager should use a CE as the basis for deciding how to best direct traffic.

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Other game-theoretic metrics exists, but are not used in this paper (see the work of Blum *et al.* [2]).

Our model incorporates a vehicle-to-infrastructure (V2I) communication protocol (see [8] for an approach to traffic management using V2I technology). We assume a V2I system that allows (a) vehicles to communicate their intended paths (e.g., turn left, go straight, turn right) and request permission; and (b) the infrastructure to perform computations and send permission approval. The Autonomous Intersection Management (AIM) project has demonstrated how an intersection management system can be designed in the case of fully autonomous vehicles [4]. We compare our algorithms for connected vehicles against AIM's reservation-based algorithm for autonomous vehicles in Section 6. The AIM group as well as Talebpour *et al.* show the advantages of game theory in modeling driver incentives and defining metrices for analysis (see [3] and [10]). Even so, Carlino *et al.* create a surrogate incentive (a currency used by vehicles) to support their game theoretic model, whereas our approach models driver incentives directly through a function of delay the driver has suffered. The work by Papadimitriou and Roughgarden in designing time-efficient algorithms for computing CE in games that yield space-efficient representations served as early theoretical reassurance that CE as a solution concept could be a computationally viable choice [7].

2 Definitions

We will first introduce some terminology and notation from the field of game theory (see, e.g., [6] for further information). A *player* is a rational entity with personal incentives. Let n denote the number of players. The ith player can choose its actions from a *strategy set*, denoted S_i. Let $S = S_1 \times \cdots \times S_n$ denote the *strategy space*. An element $s = (s_1, \ldots, s_n) \in S$ is a *strategy profile*. When dealing with the ith player, it will be convenient to separate the ith component of the profile from the others. Letting $s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$, we will represent s as (s_{-i}, s_i). Define S_{-i} analogously.

Given a strategy space S, a *utility function* $u : S \to \mathbb{R}$ is a function, where $u(s)$ intuitively measures the "benefit" of playing the strategy profile s.

An n-player *game* $G = (S, U)$ is defined to be a strategy space $S = S_1 \times \cdots \times S_n$ and a set $U = \{u_1, \ldots, u_n\}$ of utility functions. S_i and u_i denote the strategies and utilities associated with player i.

The *Nash equilibrium* is the traditional standard for analyzing the choices of rational players in game theory. A generalization of the Nash equilibrium, which more accurately reflects situations where a centralized controller (in our case, the traffic system) can recommend strategy choices, is the *correlated equilibrium*. This is defined to be a probability distribution characterized by the random vector $X = (X_1, \ldots, X_n)$ taking on values in S such that for every player i and for every pair of strategy choices $s_i, s_i' \in S_i$,

$$\sum_{s_{-i}\in S_{-i}} \big(u_i(s_{-i},s_i) - u_i(s_{-i},s'_i)\big)\, Pr[X = (s_{-i},s_i)] \;\geq\; 0, \tag{1}$$

where $X = (X_{-i}, X_i)$ is defined analogously to $s = (s_{-i}, s_i)$. That is, X satisfies the property that no player can increase its expected utility by unilaterally deviating from X's value. In contrast to the Nash equilibrium, which requires exponential time to compute in the worst case, a CE can be computed in polynomial time [7]. Given a global utility function u, a CE maximizing u in expectation is called an *optimal correlated equilibrium*.

A useful concept for expressing games succinctly is a *graphical game*. Each player's utility depends only upon a subset of the other players, and these dependencies are represented as a graph in which an edge (u,v) exists when the utilities of players u and v depend upon each other. This property, which will hold for our formulations, reduces the complexity of a game's description, and hence decreases the time needed to compute CE. A treatment of CE in graphical games is given in [5].

3 Overview of Our Traffic Model

Each vehicle has an intended path through a single intersection. If two paths intersect, these vehicles should avoid entering the intersection at the same time (see Fig. 1(a)). We model each path as a node in a *conflict graph*, where two intersecting paths are connected by an edge (see Fig. 1(b)).

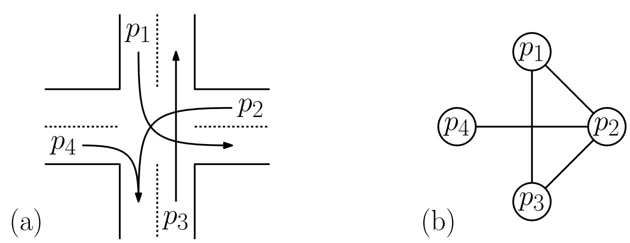

Fig. 1 (a) Configuration of paths and (b) conflict graph

The paths serve as convenient surrogates for representing the vehicles' incentives. Therefore, the paths are the players in the game for this model. At any time, a path can either be "on" or "off", corresponding to whether vehicles traveling along the path are permitted through the intersection. It is the job of a system called the *intersection manager* to decide from one time interval to the next which paths are to be switched on and which are switched off. With an appropriate discretization of time, the intersection manager can alter its recommendation at periodic intervals.

Given paths $p_1, \ldots, p_n$ through an intersection, each path has the strategy set $\{0,1\}$, where 0 corresponds to "off" (or "stop") and 1 to "on" (or "proceed"). Hence, the strategy space for this model is simply $\{0,1\}^n$. Let $\delta(p_i)$ be a positive number representing the sum of delays for vehicles traveling along p_i. Define p_i's utility $u_i : \{0,1\}^n \to \mathbb{R}$ as

$$u_i(s) \;=\; \begin{cases} 0 & \text{if } s_i = 0 \\ \delta(p_i) & \text{if } s_i = 1 \text{ and no other “on” path intersects } p_i \\ -f(p_i) & \text{otherwise,} \end{cases}$$

where f is some appropriate positively valued function. A collision is an undesirable event, justifying the negative utility for crossing paths. (Traffic management systems like AIM [4] may allow vehicles with intersecting trajectories to enter the intersection at the same time, but it assumes complete control of vehicle motions, thus preventing collisions from occurring.) With the intention to provide a balance between fairness and delay minimization, it seems natural to have the intersection manager calculate an optimal CE with respect to the global utility $u = \sum_i u_i$ then sample from this distribution for permitting vehicles through the intersection.

4 Correlated Equilibria in Independent Set Games

An *independent set* in a graph is any subset of vertices that share no edge in common. Given the goal of avoiding collisions, it is natural that the strategies suggested by an intersection manager should correspond to independent sets in the conflict graph. Based on this principle, we introduce the concept of an *independent set game*. The possible profile vectors $s = (s_1, \ldots, s_n) \in \{0,1\}^n$ are in one-to-one correspondence with subsets of vertices of the conflict graph G, where vertex i is included in the subset if and only if $s_i = 1$. We can assert $s \in IS(G)$, where $IS(G)$ denotes the collection of independent sets in G. We will denote vertex i's neighborhood as $N_i = \{j \mid (i,j) \text{ is an edge in } G\}$. Finally, we will write $N_i(s) = \sum_{j \in N_i} s_j$ to mean the sum of vertex i's neighbors in s. Note that $N_i(s) = 0 \Leftrightarrow \forall_{j \in N_i}\, s_j = 0$ and $N_i(s) > 0 \Leftrightarrow \exists_{j \in N_i}\, s_j = 1$. Vertex i's utility can therefore be expressed as

$$u_i(s) = \begin{cases} 0 & \text{if } s_i = 0 \\ a_i & \text{if } s_i = 1 \text{ and } N_i(s) = 0 \\ -b_i & \text{if } s_i = 1 \text{ and } N_i(s) > 0, \end{cases}$$

where $a_i, b_i > 0$.

It is clear that our traffic model satisfies the definition of an independent set game. Given an intersection conflict graph, we will formulate the behavior of vehicles through an intersection as a graphical game. This is valid since, by the definition of u_i in the model, the utility of path p_i depends only on the decisions of its neighbors in the conflict graph that is, the intersecting paths.

As argued in Section 3, it will be useful to characterize the CEs and optimal CEs that can arise in an independent set game. We consider two types of independent set games. The first type, a *finite independent set game*, applies when $b_i < \infty$. The second type, an *infinite independent set game*, is the limiting case $b_i \to \infty$ as an extension of the finite case. Formally, given any positive sequence $b_i(n)$ for $n \geq 1$ such that $\lim_{n\to\infty} b_i(n) = \infty$, we require that there exists an N such that the CE constraints hold for all $b_i(n)$ where $n \geq N$. For the traffic application, this is a natural condition where a collision between two vehicles is assigned an infinitely high cost.

4.1 Finite Case

We first consider the finite case. By the definition of a CE we have the following.

Lemma 1. *A random vector X is a correlated equilibrium of a finite independent set game if and only if X satisfies*

$$Pr[X_i = 1, N_i(X) = 0] \ \geq \ \frac{b_i}{a_i} \cdot Pr[X_i = 1, N_i(X) > 0] \tag{2}$$

$$Pr[X_i = 0, N_i(X) = 0] \ \leq \ \frac{b_i}{a_i} \cdot Pr[X_i = 0, N_i(X) > 0]. \tag{3}$$

Proof. For each vertex i, there are two nontrivial CE constraints on X (following from Eq (1)).

$$\sum_{s_{-i}\in S_{-i}} (u_i(s_{-i},1) - u_i(s_{-i},0))\, Pr[X = (s_{-i},1)] \ \geq \ 0$$

$$\sum_{s_{-i}\in S_{-i}} (u_i(s_{-i},0) - u_i(s_{-i},1))\, Pr[X = (s_{-i},0)] \ \geq \ 0.$$

By unfolding the definition of u_i, the first inequality becomes

$$\begin{aligned}
\sum_{s_{-i}\in S_{-i}} (u_i(s_{-i},1) - u_i(s_{-i},0))\, Pr[X = (s_{-i},1)] \ &\geq \ 0 \\
\sum_{s_{-i}\in S_{-i}} u_i(s_{-i},1) Pr[X = (s_{-i},1)] \ &\geq \ 0 \\
a_i \sum_{s_{-i}:N_i(s_{-i})=0} Pr[X = (s_{-i},1)] - b_i \sum_{s_{-i}:N_i(s_{-i})>0} Pr[X = (s_{-i},1)] \ &\geq \ 0 \\
a_i Pr[X_i = 1, N_i(X) = 0] - b_i Pr[X_i = 1, N_i(X) > 0] \ &\geq \ 0,
\end{aligned}$$

and hence Eq (2) holds. By analogous algebra, Eq (3) also holds.

Given that b_i denotes the penalty incurred by allowing the possibility of a collision, it is natural to ask whether there is some sufficiently large finite value of b_i that guarantees that X only evaluates to independent sets? Does an answer to this question require non-trivial restrictions on G? Even though it seems natural to conjecture that such non-trivial requirements are necessary, the answer is, surprisingly, contrary.

Theorem 1. *If a finite independent set game with graph G has at least one edge, then there exists a correlated equilibrium X for which* $Pr[X = s] > 0$ *for some* $s \notin IS(G)$.

Proof. We will explicitly construct a distribution with the desired property and show that it satisfies Lemma 1. Let S_{MIS} be a finite set of independent sets that satisfies the property: For each $v \in G$, $v \in s$ for some $s \in S_{MIS}$. Let $m = |S_{MIS}|$. Define

$$\frac{b}{a+mb} \;=\; \max_{1 \le i \le m} \frac{b_i}{a_i + mb_i}.$$

Note that this implies

$$\frac{b}{a} \;=\; \max_{1 \le i \le m} \frac{b_i}{a_i}.$$

The following distribution is a CE.

$$\forall s \in S_{MIS},\; Pr[X = s] \;=\; \frac{b}{a+mb}$$
$$Pr[X = (1,\ldots,1)] \;=\; \frac{a}{a+mb}.$$

This is indeed a valid distribution since the values are all positive and

$$\frac{a}{a+mb} + \sum_{s \in S_{MIS}} \frac{b}{a+mb} \;=\; 1.$$

First we check that Eq (3) is satisfied. Note that if $X_i = 0$, then $X \in S_{MIS}$ and hence $(X_{-i}, 1) \notin IS(G)$ by maximality of X. Thus, $Pr[X_i = 0, N_i(X) = 0] = 0$ for all i, implying Eq (3) is satisfied. To see that Eq (2) is satisfied, observe

$$\begin{aligned} Pr[X_i = 1, N_i(X) = 0] \ge Pr[X = s] \;&=\; \frac{b}{a+mb} \;=\; \frac{b}{a}\frac{a}{a+mb} \\ &= \frac{b}{a} Pr[X = (1,...,1)] \;=\; \frac{b}{a} Pr[X_i = 1, N_i(X) > 0] \\ &\ge \frac{b_i}{a_i} Pr[X_i = 1, N_i(X) > 0] \end{aligned}$$

where s is any $s \in S_{MIS}$ with $i \in s$.

From Theorem 1, we see that it is not the case that there are a small number of adversarial Gs having some CE that recommends a non-independent set with positive probability. In fact, *all* graphs that contain a non-independent set are associated with such a CE. Hence, no finite value b_i can yield Theorem 2, even when imposing non-trivial restrictions on G.

Example Application of Theorem 1

Suppose G is as in Fig. 2 and $b_i = 2a_i$ for all i. Then, the theorem says we have a distribution over the sets in Fig. 3. In particular, setting $Pr[X = (0,1,0)] = Pr[X = (1,0,1)] = 2/5$ and $Pr[X = (1,1,1)] = 1/5$ – as in the proof of the theorem – guarantees a CE. This is true even through $1/5$ of the time the system recommends a non-independent set.

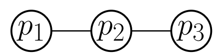

Fig. 2 An example graphical game graph.

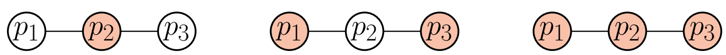

Fig. 3 Strategy vectors with non-zero probabilities in a particular CE.

4.2 Infinite Case

We now consider the infinite case. Our analysis of this case will make use of a property of collections of independent sets. A collection of independent sets $\{V_1, \ldots, V_n\}$ is said to be *mutually maximal* if, for all V_i and vertices v, it holds that $v \notin V_i$ implies there is some V_j such that $V_j \cup \{v\}$ is not independent. Note that $\{V_1\}$ is mutually maximal if and only if V_1 is a maximal independent set. See Fig. 4 for an example of a set of mutually maximal independent sets in which no independent set is maximal.

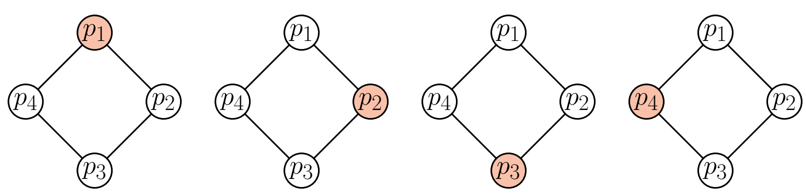

Fig. 4 Mutually maximal independent sets where no single independent set is maximal.

Given this concept, we can characterize the distributions that are CE.

Theorem 2. *A random vector X is a correlated equilibrium of an infinite independent set game if and only if X evaluates to mutually maximal independent sets in the game graph.*

Proof. From Lemma 1, we require for each vertex i and positive integer $n \geq N$

$$Pr[X_i = 1, N_i(X) = 0] \;\geq\; \frac{b_i(n)}{a_i} Pr[X_i = 1, N_i(X) > 0]$$
$$Pr[X_i = 0, N_i(X) = 0] \;\leq\; \frac{b_i(n)}{a_i} Pr[X_i = 0, N_i(X) > 0],$$

which is equivalent to

$$Pr[X_i = 1, N_i(X) > 0] \;\leq\; \frac{a_i}{b_i(n)} Pr[X_i = 1, N_i(X) = 0]$$
$$Pr[X_i = 0, N_i(X) > 0] \;\geq\; \frac{a_i}{a_i + b_i(n)} Pr[X_i = 0],$$

which is then equivalent to

$$Pr[X_i = 1, N_i(X) > 0] \;=\; 0 \tag{4}$$
$$Pr[N_i(X) > 0 | X_i = 0] \;>\; 0 \qquad \text{or} \qquad Pr[X_i = 0] \;=\; 0. \tag{5}$$

Unlike the finite case, the infinite case always disallows non-independent sets, since Eq. (4) dictates that a CE can only recommend independent sets in G. Eq. (5) dictates that a CE must either recommend some vector that, upon including i, is not an independent set or else always recommend that i be included. Together, the inequalities dictate that X forms a CE if and only the set of s's with positive probabilities are mutually maximal independent sets.

In the context of the example in Fig. 1, a CE could be a probability distribution over the maximal independent sets in Fig. 5. But, the mutually maximal independent sets need not be individually maximal, as Fig. 4 shows.

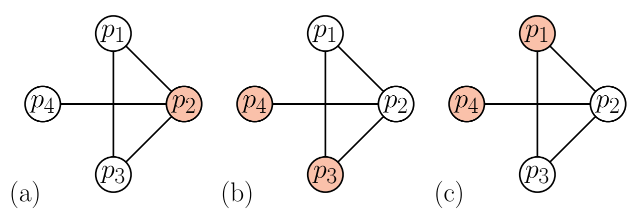

Fig. 5 Maximal independent sets of conflict graph in Fig. 1(b)

4.3 Optimal CE

After exploring the nature of equilibria in independent set games, one may wonder how the optimal CE behave. We will set the global utility function as $u = \sum_i u_i$ and weight each vertex i in the graph by a_i. The ensuing result is quite interesting and holds for both finite and infinite independent set games.

Theorem 3. *A random vector X is an optimal correlated equilibrium of an independent set game with respect to the global utility $u = \sum_i u_i$ if and only if X evaluates to maximum-weight independent sets in the weighted game graph.*

Proof. First, consider a finite independent set game with some equilibrium represented by the random vector X. Then, the expected global utility is

$$E[u(X)] = \sum_i E[u_i(X)]$$

by linearity of expectation. Theorem 1 says that we could have non-independent sets in the finite case. For any $s \notin IS(G)$, we can remove vertices which contribute negatively to $u(s)$ from s until the remaining vertices $s' \subset s$ form a maximal independent set. $u_i(s) \leq u_i(s')$ since

$$u_i(s') - u_i(s) = \begin{cases} a_i + b_i & \text{if } s_i = 1, s'_i = 1, N_i(s) > 0 \\ 0 & \text{if } s_i = 1, s'_i = 1, N_i(s) = 0 \\ b_i & \text{if } s_i = 1, s'_i = 0, N_i(s) > 0 \\ 0 & \text{if } s_i = 1, s'_i = 0, N_i(s) = 0 \\ 0 & \text{if } s_i = 0, s'_i = 0 \end{cases}$$

for every i. Therefore, we can form a new equilibrium X' where we set $Pr[X' = s] = 0$ and $Pr[X' = s'] = Pr[X = s'] + Pr[X = s]$ and, by the argument above, this results in

$$E[u(X)] = \sum_i E[u_i(X)] \leq \sum_i E[u_i(X')] = E[u(X')] \tag{6}$$

by applying the inequality to each i. Therefore, we can restrict our domain to independent sets without loss of generality. Suppose that s is an independent set and $s' \supset s$ is a maximal independent set. Then $u_i(s) \leq u_i(s')$ since

$$u_i(s') - u_i(s) = \begin{cases} 0 & \text{if } s_i = 1, s'_i = 1 \\ a_i & \text{if } s_i = 0, s'_i = 1 \\ 0 & \text{if } s_i = 0, s'_i = 0 \end{cases}.$$

So we can restrict our domain even further to maximal independent sets (denoted $MIS(G)$) as in the argument above for independent sets. $u(s)$ is exactly the sum of the a_i weights when $s \in MIS(G)$. By definition, any maximal independent set has sum of weights at most equal to the sum of weights in a maximum weight independent set (denoted $MWIS(G)$) which gives us our last restriction on the domain of X. Let $M = u(s)$ for $s \in MWIS(G)$ (which is constant over $MWIS(G)$). Then,

$$\begin{aligned} E[u(X)] &= \sum_{s \in S} u(s) Pr[X = s] \leq \sum_{s \in MWIS(G)} u(s) Pr[X' = s] \\ &= M \sum_{s \in MWIS(G)} Pr[X' = s] = M \end{aligned}$$

for an appropriate transformation $X \mapsto X'$ to restrict the domain to $MWIS(G)$ and increase the utility as above.

For the infinite case, recall from Theorem 2 the fact that the CE domain is already restricted to independent sets, and so the above argument holds.

5 Independent Sets and Non-Crossing Matchings

In this section we consider how to compute maximum-weight independent sets in the conflict graph. In general, computing maximum independent sets is known to be hard, even to approximate (see, e.g., [1]). But here, n is practically bounded by the size of intersections, and we can exploit special properties of these graphs to obtain efficient solutions. We present two algorithms. The first is a simple and efficient greedy heuristic, which does not necessarily generate an optimal solution. The second, which is based on dynamic programming, guarantees an optimal solution, but it has higher computational complexity.

A single intersection is given, which is modeled as a central resource that is surrounded by a circular collection of lanes, radiating outwards from the intersection. Each lane is directed either into the intersection (*in-lane*) or out from the intersection (*out-lane*). For example, in Fig. 6(a), the lanes with odd indices are in-lanes and the lanes with even indices are out-lanes. A graph G is given, each of whose edges represents a path along which vehicles wish to travel. Its nodes are the lanes and each edge p consist of one in-lane and one out-lane (see Fig. 6(b)). A *matching* M is a subset of edges of G, such that each node of G is an endpoint of at most one edge of M. A matching is *non-crossing* if the path associated with the edges of the matching do not conflict with each other (see Fig. 6(c)). The *weight* of a matching is $u(M)$ (valid since non-crossing matchings correspond to independent sets). Given G and the δ values for its paths, it is easy to see that an independent set of maximum weight in the conflict graph is equivalent to a non-crossing matching M that maximizes $u(M)$ in the intersection graph.

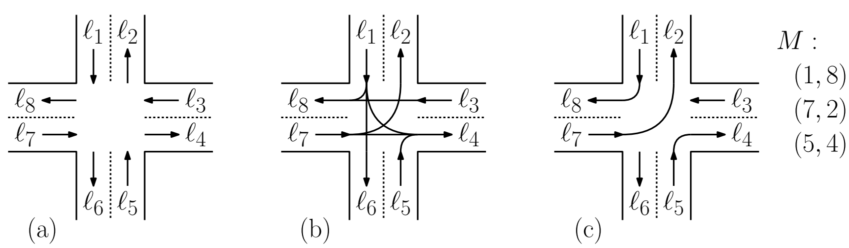

Fig. 6 An intersection and a non-crossing matching

5.1 Greedy Algorithm

We first present a simple greedy heuristic to determine which paths to admit (see Algorithm 1). Let P be the set of all paths through the intersection (of the form (i,j) as in Fig. 6). Recall that for each path $p \in P$, $\delta(p)$ denotes the summed delays of vehicles traveling along p. A simple greedy heuristic is to sort P in decreasing order of $\delta(p)$, and include a path p in the non-crossing matching if it does not cross any previously selected path. In order to avoid gridlock, we perform this step twice; In the first traversal, we only select paths p for which there exists a vehicle at the head of its queue and is traveling along p.

Algorithm 1: Greedy heuristic for a maximum-weight independent set

```
Greedy(P[n])
    M ← ∅;
    P′ ← sort P by delay;
    foreach (p ∈ P′) do
        if (existsFrontVehicle(p) ∧ doesNotCross(p,M)) then
            M ← M ∪ {p};
    foreach (p ∈ P′) do
        if doesNotCross(p,M) then
            M ← M ∪ {p};
```

The function *doesNotCross*(p,M) returns true if p does not cross any of the paths in M and returns false otherwise. This algorithm clearly takes polynomial time (no longer than $O(n^2)$ for $n = |P|$) and produces a non-crossing matching. Although, it is not necessarily a maximum matching since a locally optimum choice to include some p in M may exclude a later choice that results in a globally optimal matching. Regardless, small inefficiencies in the construction of M are not detrimental since the system is, in a certain sense, "self-correcting" due to the dynamic changes in vehicle delays.

5.2 Dynamic Programming Algorithm

We next present a more sophisticated dynamic-programming algorithm. While our dynamic-programming algorithm is slower than the greedy heuristic, it is guaranteed to compute an independent set of maximum weight. The intersection is modeled as a graph whose vertices correspond to the entry-exit points of the lanes, and edges correspond to paths. Define $G_{i,j}$ to be the induced subgraph consisting of the vertices from i to j in clockwise order about the intersection. Define $W[i,j]$ to be the weight of the maximum non-crossing matching on this subgraph. The dynamic-programming presented in Algorithm 2 computes $W[i,j]$ by considering all possible ways of connecting i to another vertex k in the $[i,j]$ interval, and then recursively solving the two subproblems, from $[i+1,k-1]$ and $[k+1,j]$.

Algorithm 2: DP algorithm for a maximum-weight non-crossing matching

```
ComputeWeight(δ[n,n])
    for (d ← 1 to n − 1) do                          // length of the vertex interval
        for (i ← 0 to n − d − 1) do                  // first vertex of the interval
            j ← i + d;                               // last vertex of the interval
            wt ← W[i+1, j];
            G′ ← induced(G, i, j);                   // induced subgraph on [i, j]
            foreach (k adjacent to i in G′) do       // try path (i, k)
                curr ← δ[i,k] + W[i+1, k−1] + W[k+1, j];
                wt ← max(wt, curr);
            W[i, j] ← wt;
```

6 Experimental Results

We wrote an intersection simulator that implements the greedy and DP algorithms from the previous section, and we compare the queueing behavior in our system versus other systems. Define $Q_{T,L}$ as the number of vehicles with zero velocity in lane L after simulator time step T. Then, let $Q_T = \max_L Q_{T,L}$. We define the *average max queue length* as

$$\bar{Q}_T \;=\; \frac{1}{T}\sum_{t\leq T} Q_t .$$

Fig. 7 shows the effect of traffic flow on $\bar{Q}_{100000}$ corresponding to 33 minutes of simulated physical time. Note that the $\bar{Q}_{100000}$ values approach an asymptote due to the lane capacity in the simulations. Looking first at the stop sign versus a four phase traffic light, we confirm the common knowledge that stop signs are more effective than traffic lights for low traffic flows, but traffic lights are more effective for higher traffic flows (in our case, the intersection occurs around 120 vehicles per hour per lane). However, our greedy algorithm outperforms both of these systems, and is able to handle much higher traffic flows without filling the lane capacities. While the AIM reservation-based protocol from UT Austin [4] can handle higher traffic flows, our system supports the weaker assumption of connected vehicles as opposed to the fully autonomous vehicles of AIM. Finally, the optimal CE minimizes delay while simultaneously ensuring a certain degree of fairness. Smith and Gali [9] support the notion that it is often desirable to constrain delay minimization in traffic networks by some notion of fairness.

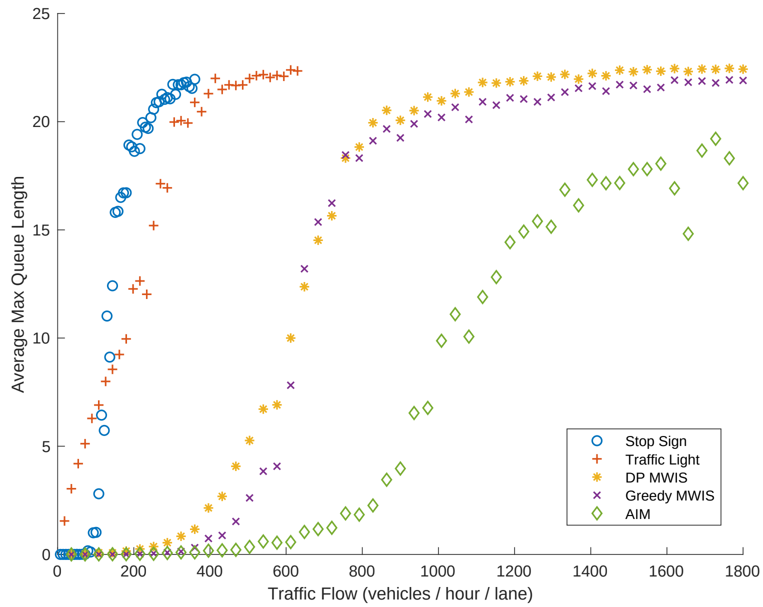

Fig. 7 Effect of traffic flow on average max queue length.

7 Conclusion

We showed that our game-theoretic traffic model based on delay and safety reduces to an independent set game. Theorem 1 and Theorem 2 show that if one wants to prohibit the possibility that vehicles could accept an accident with small probability, then the model needs to assign an infinite cost to an accident. Theorem 3 shows that regardless of the choice of cost, a coordinating agent can select a maximum-weight independent set and guarantee an optimal CE. Our experimental simulations show that our approach for connected vehicles allows more throughput than standard control mechanisms yet less throughput than AIM's reservation-based protocol for fully autonomous vehicles. Therefore, our approach provides practical improvement over current methods while vehicles on the road are not yet fully autonomous.

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