Problems. In each of the following problems you are given a language and are asked to produce a regular expression and/or finite automaton for the language. In some cases you are asked to give “either” a DFA or regular expression (your choice) and in other cases to give “both” a DFA and regular expression. When writing regular expressions, use the shorthand $\epsilon$ to denote the empty string. Write DFA’s in the form of a transition diagram. The underlying alphabet is $\Sigma = \{a, b\}$.  

The notation $#a(w)$ appearing below means the number of $a$’s occurring in string $w$. For example, $#a(bbaba) = 2$.

Note that in all the DFAs shown, missing transitions signify a transition to a dead state (a non-final state with transitions back to itself on all elements of the input alphabet).

1. (Either DFA or Reg. Exp) $\{w | w \text{ begins with } abab \}$. 

![DFA Diagram]
2. (Either) \( \{ w | w \text{ ends with } abab \} \).

3. (Either) \( \{ w | w \text{ begins with } ab \text{ and ends with } ba \} \).
(Note: The string \( aba \) is in this language!)

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4. (Either) \( \{ w \mid \text{either } \#a(w) \text{ is divisible by } 3 \text{ or } w \text{ begins with } bbb \} \).

5. (Either) \( \{ w \mid \#a(w) \equiv 2 \pmod{5} \} \).
   (Recall that \( i \equiv j \pmod{k} \) if and only if \( i - j \) is divisible by \( k \)).
6. (Either) \( \{ w | \#a(w) \equiv 1 \pmod{3} \text{ and } \#b(w) \text{ is odd} \} \).

7. (Either) \( \{ w | \#a(w) \text{ is even or } |w| \text{ is even} \} \).
8. (Both DFA and Reg. Exp) \( \{ w | \text{aaa is a substring of } w \} \).

\[
(a \mid b)^* \ (\text{aaa}) \ (a \mid b)^*
\]

9. (Both) \( \{ w | \text{aaa is not a substring of } w \} \).

\[
(b^* \ (e \mid a \ | \ a a) \ b)^* \ (e \mid a \ | \ a a)
\]
10. (Either) \( \{ w \mid w \text{ contains exactly one occurrence of the substring } aaaS \} \).
(Note: the string validation has two occurrences of aaaS!)

11. (DFA only) \( \{ w \mid \text{neither } aa \text{ nor } bb \text{ is a substring of } w \} \).