Allocating a Discrete Set Of Goods

William Gasarch-U of MD

AB Methods

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An Estate

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C's Will

In C's will she leaves her children A and B.

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A House (H), A Car (C), A Boat (B)

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In *C*'s will she leaves her children *A* and *B*.

A House (H), A Car (C), A Boat (B)

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Money: $5000, (M)

An Estate

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Money: \$5000, (M)

5 items total. 4 are discrete. The money is essentially continuous.

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A didn't say who gets what!

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5 items total. 4 are discrete. The money is essentially continuous.

A didn't say who gets what!

How do *A* and *B* decide who gets what? Discuss?

Alternate Methods

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A and B pick one item at a time.

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They use ABABA.

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They use ABABA. Discuss PROS and CONS.

PRO Simple. A and B do not need formal valuations.

CON A has two advantages: goes first and goes 3 times.

Discuss how to modify to make more fair.

Snake Method

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A and B pick one item at a time.

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They could alternate: ABBAB.

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Caveat How fair depends on A's and B's valuations.

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Caveat How fair depends on A's and B's valuations.

Oh We need a notion of valuation.

Valuations

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Def Let $I_1, \ldots, I_m$ be a set of items. A **valuation** is a function that assigns to each item a natural number such that the sum of the numbers is 100.

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Example A House (H), A Car (C), A boat (B),
A Picture of Alice with Taylor Swift signed by Taylor Swift (P)
Money \$5000 (M)

A's valuation:
$Val(H) = 30$
$Val(C) = 20$
$Val(B) = 10$
$Val(P) = 30$ (Alice is a swifty!)
$Val(M) = 10$

Alternate-Valuation Protocol

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A takes an item.

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A takes an item.

B take an item. If B still has less than A, take another.

More generally, whoever currently has less points takes next item.

Example

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Item	V_A	V_B
House	30	50
Car	20	15
Boat	10	15
Picture	30	0
Money	10	20

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Item	V_A	V_B
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We keep track of $A : B$

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House	30	50
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A takes house. 30:0

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Item	V_A	V_B
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Car	20	15
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Picture	30	0
Money	10	20

We keep track of $A : B$

A takes house. 30:0

B takes money. 30:20

Example

Item	V_A	V_B
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Car	20	15
Boat	10	15
Picture	30	0
Money	10	20

We keep track of $A : B$

A takes house. 30:0

B takes money. 30:20

B takes car. 30:35

Example

Item	V_A	V_B
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Car	20	15
Boat	10	15
Picture	30	0
Money	10	20

We keep track of $A : B$

A takes house. 30:0

B takes money. 30:20

B takes car. 30:35

A takes picture. 60:35

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Car	20	15
Boat	10	15
Picture	30	0
Money	10	20

We keep track of $A : B$

A takes house. 30:0

B takes money. 30:20

B takes car. 30:35

A takes picture. 60:35

B takes boat. 60:50.

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Item	V_A	V_B
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Boat	10	15
Picture	30	0
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A takes house. 30:0

B takes money. 30:20

B takes car. 30:35

A takes picture. 60:35

B takes boat. 60:50.

Unfair!

Example

Example

Item	V_A	V_B
House	30-A	50
Car	20	15-B
Boat	10	15-B
Picture	30-A	0
Money	10	20-B
	60	50

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Item	V_A	V_B
House	30-A	50
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Picture	30-A	0
Money	10	20-B
	60	50

Discuss how to do a better protocol

Example

Item	V_A	V_B
House	30-A	50
Car	20	15-B
Boat	10	15-B
Picture	30-A	0
Money	10	20-B
	60	50

Discuss how to do a better protocol

B wants the house more than A. So B should get the house.

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Discuss how to do a better protocol

B wants the house more than A. So B should get the house.

We expand on this idea.

Adjusted Winner Protocol

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First Step of Adjusted Winner

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For each item give it to the person who values it the most.

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Item	V_A	V_B
House	30	50-B
Car	20-A	15
Boat	10	15-B
Picture	30-A	0
Money	10	20-B
	50	85

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Discuss how to **adjust** so that both have the same.

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After the first step we have the following:

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Gap is $85 - 50 = 35$.

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	50	85

Gap is $85 - 50 = 35$.

1) If B gave boat to A would help close the gap.

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Item	V_A	V_B
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Car	20-A	15
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2) The money is continuous so we can split it.

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If we transfer x dollars from A to B then

A gains $10x$ points.

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So need $60 + 10x = 70 + 20x$.

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So B should give $\frac{1}{3}$ of the money to A.

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Item	V_A	V_B
House	30	50-A
Car	20-A	15
Boat	10-A	15
Picture	30-A	0
Money	$\frac{1}{3}10 = \$1666.67$-A	$\frac{2}{3}20 = \$3333.33$-B
	$63.33\cdots$	$63.33\cdots$

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How to split the money IRL

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How to split the money IRL

Alice get \$1666.67

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How to split the money IRL

Alice get \$1666.67

Bob gets \$3333.33

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	$63.33\cdots$	$63.33\cdots$

How to split the money IRL

Alice get \$1666.67

Bob gets \$3333.33

Bob should not complain that he is getting 1 cent less than $\frac{2}{3}$ of the \$5000.

Second Example of Adjusted Winner

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Item	V_A	V_B
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Car	15	15-B
Boat	10	15-B
Picture	35-A	0
Money	12	20-B
	35	100

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Gap is 65.

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Picture	35-A	0
Money	12	20-B
	35	100

Gap is 65.

Discuss what to do so that both have the same.

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1) First look at ties where the B got the good. If giving item to A narrows the diff then do it.

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Here it does.

B should give A the Car.

After Give Alice Car

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Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10	15-B
Picture	35-A	0
Money	12	20-B
	50	85

After Give Alice Car

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10	15-B
Picture	35-A	0
Money	12	20-B
	50	85

Gap is 35.

After Give Alice Car

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10	15-B
Picture	35-A	0
Money	12	20-B
	50	85

Gap is 35.

if B gave A all the money then would be 62:65, still uneven. So look at other items first.

After Give Alice Car

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10	15-B
Picture	35-A	0
Money	12	20-B
	50	85

Gap is 35.

if B gave A all the money then would be 62:65, still uneven. So look at other items first.

2) Can A give B something to narrow the gap?

After Give Alice Car

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10	15-B
Picture	35-A	0
Money	12	20-B
	50	85

Gap is 35.

if B gave A all the money then would be 62:65, still uneven. So look at other items first.

2) Can A give B something to narrow the gap?

Yes-A can give B the Boat.

After Bob Gives Alice Boat

After Bob Gives Alice Boat

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10-A	15
Picture	35-A	0
Money	12	20-B
	60	70

After Bob Gives Alice Boat

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10-A	15
Picture	35-A	0
Money	12	20-B
	60	70

Gap is 10

After Bob Gives Alice Boat

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10-A	15
Picture	35-A	0
Money	12	20-B
	60	70

Gap is 10

Now we use money.

After Bob Gives Alice Boat

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10-A	15
Picture	35-A	0
Money	12	20-B
	60	70

Gap is 10

Now we use money. $60 + 12x = 70 - 20x$.

After Bob Gives Alice Boat

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10-A	15
Picture	35-A	0
Money	12	20-B
	60	70

Gap is 10

Now we use money. $60 + 12x = 70 - 20x$. $32x = 20$.

After Bob Gives Alice Boat

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10-A	15
Picture	35-A	0
Money	12	20-B
	60	70

Gap is 10

Now we use money. $60 + 12x = 70 - 20x$. $32x = 20$. $x = \frac{5}{8}$.

After Bob Gives Alice Boat

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10-A	15
Picture	35-A	0
Money	12	20-B
	60	70

Gap is 10

Now we use money. $60 + 12x = 70 - 20x$. $32x = 20$. $x = \frac{5}{8}$.

$\frac{5}{8} \times 5000 = 3125$.

After Bob Gives Alice Boat

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10-A	15
Picture	35-A	0
Money	12	20-B
	60	70

Gap is 10

Now we use money. $60 + 12x = 70 - 20x$. $32x = 20$. $x = \frac{5}{8}$.

$\frac{5}{8} \times 5000 = 3125$. No fractions! Thats rare.

Final Allocation

Final Allocation

Item	V_A	V_B
House	28	50-B
Car	15-A	15
Boat	10-A	15
Picture	35-A	0
Money	$\frac{5}{8} \times 12 = \$3125$-A	$\frac{3}{8}20 = \$1875$-B
	67.5	67.5

PROS and CONS of AW

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CONS

If A knows B's valuation she can use that to lie and do better.

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If A knows B's valuation she can use that to lie and do better.
If A and B are antagonistic then may lie out of spite.

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Simple, Proportional, Envy-Free.

Pareto Optimal: No allocation can make one person better off without making the other one worse off.

CONS

If A knows B's valuation she can use that to lie and do better.

If A and B are antagonistic then may lie out of spite.

Example A doesn't care about the car but wants it because B wants it.

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Have historians done this? Yes, but more as part of a more complex argument.