Cut and Choose (cc)

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Credit Where Credit is Due

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In the Bible Abraham says to Lot

Do you want the West or East Part of the Land (Cutting).

Lot says

I'll take the East Part (Choosing).

Format For Protocols

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What the players **are required** to do

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1) *A* cuts the cake in half (equal in her eyes)

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Example for Cut and choose:

1) A cuts the cake in half (equal in her eyes)

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When we give a protocol we give both
What the players **are required** to do
What the players **should do** in parenthesis.

Example for Cut and choose:

1) A cuts the cake in half (equal in her eyes)

2) B picks one of those pieces (the bigger one in his eyes)

We will always show that if a player cheats (does not follow what they should do) then they may end up with **less** than if they didn't cheat.

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1) Two player.

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2) Dividing a cake (a continuous good).

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For this talk **Protocol** always means:

1) Two player.

2) Dividing a cake (a continuous good).

3) They may have different tastes (geometry not helpful).

General Procedure

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A and B want to divide a cake

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Scenario: A creates (P_1, P_2): $V_A(P_1) < \frac{1}{2}$ and $V_A(P_2) > \frac{1}{2}$.

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If B takes P_2 then
A has P_1 and $< \frac{1}{2}$. A has LESS than if honest.

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Cheating would mean taking the smaller piece. Obviously a bad idea.

The protocol is cheat-proof. Will assume from now on that both players are honest.

Proportional and Envy Free

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A's View: Both get $\frac{1}{2}$.

B's View: B gets tied or bigger piece.

Neither player is envious.

DISCUSSION

DISCUSS PROS AND CONS OF PROTOCOL

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1) Proportional, Envy Free.

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2) Players need not have precise valuation.

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3) Works for ANY valuations.

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1) Proportional, Envy Free.
2) Players need not have precise valuation.
3) Works for ANY valuations.
4) Pieces are continuous.

CON

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Not Equitable:

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Scenario:

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A cuts cake (P_1, P_2)
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A has P_1, so has $\frac{1}{2}$.

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A has P_1, so has $\frac{1}{2}$.

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So not equitable.

Is there an Equitable Protocol?

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For now we look at **approximate** equitable.

ϵ-Equitable

Def A division (P_1, P_2) where A gets P_1 and B gets P_2 is **ϵ-Equitable** if

$$|V_A(P_1) - V_B(P_2)| < \epsilon.$$

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Answer on next slide.

Yes We Can

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If $a > b$ then A gets $(a, 1]$ and B gets $[0, b)$.

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3) They both have $1/2$. We assume $a \le b$, other case similar.

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5) $V_A([a, b]) < \epsilon$ & $V_B([a, b]) < \epsilon$: cut-choose on $[a, b]$-DONE.
6) $V_A([a, b]) \ge \epsilon$ or $V_B([a, b]) \ge \epsilon$: repeat with $[a, b]$.

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THEN:

A has $[0, a_n]$, B has $[b_n, 1]$.

$V_A([a_n, b_n]) < \epsilon$ and $V_B([a_n, b_n]) < \epsilon$.

No matter how $[a_n, b_n]$ is split, A and B will differ by $< \epsilon$.

Why Equitable?–I cheated on last slide

Let A and B execute the protocol. Let the sequence of A-cuts be $a_1 < a_2 < \cdots < a_n$ and the sequence of B-cuts be $b_1 > b_2 > \cdots > b_n$. **MIGHT NOT HAPPEN THAT WAY**.
Could be that $a_1 < b_1$ but $a_2 > b_2$. **Pieces might not be continuous**.
Let A_i be what A has after i iterations.
Let B_i be what B has after i iterations.

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One of the worksheet problems will be to finish this case.

DISCUSSION

DISCUSS PROS AND CONS OF PROTOCOL

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Proportional, Envy Free, Cheat proof, ϵ-equitable.

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CON

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A and B need to quantify their valuations.

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How A Can Cheat

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Scenario

	I_1	I_2	I_3	I_4	I_5	I_6
A	3	3	2	2	1	1
B	1	1	1	2	3	4

If A knows B's preferences then A will cut it:
(I_1, I_2, I_3, I_4) and (I_5, I_6).

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B will take the (I_5, I_6) which is worth 7.

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A is left with (I_1, I_2, I_3, I_4) which is worth 9.

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Scenario

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B will take the (I_5, I_6) which is worth 7.

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A used the knowledge of B's preference to cut the cake in a way that gives A more.

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Obtaining this seems very hard. We may need to drop another requirement.
A protocol is ϵ-**proportional** each player has within ϵ of $\frac{1}{2}$.

ϵ-proportional Super Cheat Proof

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Theorem For all ϵ there exists an ϵ-proportional super-cheat-proof protocol.

Phase One

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A cuts into $2L$ pieces. (Evenly)

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A and B reveal what each piece is worth.

Phase One

Let L be such that $\epsilon \leq \frac{1}{L}$.
A cuts into $2L$ pieces. (Evenly)
B cuts each piece into $2L$ pieces. (Evenly)
A and B reveal what each piece is worth.
Pieces: $p_1, \ldots, p_m$. $\mathrm{V}_A(p_i)$ be how much A values piece p_i.
$\mathrm{V}_B(p_i)$ be how much B values piece p_i. (If both follow advice: $\mathrm{V}_A(p_i), \mathrm{V}_B(p_i) \leq \frac{1}{2L}$.)

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2) Assume $q_1, \ldots, q_k$ are already defined.

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Recall: All of the p_i's are TINY to both. A and B reorder the pieces. 1) $q_1 = p_1$.

2) Assume $q_1, \ldots, q_k$ are already defined.
2a) If $\sum_{i=1}^{k} \mathrm{V}_A(q_i) \le \sum_{i=1}^{k} \mathrm{V}_B(q_i)$ then A and B find a piece p not already used such that $\mathrm{V}_B(p) < \mathrm{V}_A(p)$. Let $q_{k+1} = p$.

Phase Two

Recall: All of the p_i's are TINY to both. A and B reorder the pieces. 1) $q_1 = p_1$.

2) Assume $q_1, \ldots, q_k$ are already defined.
2a) If $\sum_{i=1}^{k} \mathrm{V}_A(q_i) \le \sum_{i=1}^{k} \mathrm{V}_B(q_i)$ then A and B find a piece p not already used such that $\mathrm{V}_B(p) < \mathrm{V}_A(p)$. Let $q_{k+1} = p$.
2b) If $\sum_{i=1}^{k} \mathrm{V}_B(q_i) < \sum_{i=1}^{k} \mathrm{V}_A(q_i)$ then A and B find a piece p not already used such that $\mathrm{V}_A(p) < \mathrm{V}_B(p)$. Let $q_{k+1} = p$.
Intuition: $q_1 \cup \cdots \cup q_k$ valued about the same by both.

Phase Three

$q_1, \ldots, q_n$ defined.
For all k, $q_1 \cup \cdots \cup q_k$ valued about the same to both.

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2) Let $P = q_1 \cup \cdots \cup q_k$. Let $Q = q_{k+1} \cup \cdots \cup q_n$.
Intuition: P and Q both valued about $\frac{1}{2}$ by both.
Now what?

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Now what?
Flip a Coin
If its **heads** then A gets P, B gets Q.
If its **tails** then B gets Q, A gets P.

ϵ-Proportional Super Cheat Proof

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1) KEY: Neither player knows who will get P and who will get Q
2) BOTH want P and Q to be about the same size.
3) Neither will cheat for fear of getting the smaller piece.
4) Even if A knows B's tastes, no benefit to cheating.

DISCUSSION

DISCUSS PROS AND CONS OF PROTOCOL

PRO

PRO

PRO

PRO

PRO

1) ϵ-Proportional, Super-Cheat proof

PRO

PRO

1) ϵ-Proportional, Super-Cheat proof

2) Works for ANY valuations.

PRO

PRO

1) ϵ-Proportional, Super-Cheat proof

2) Works for ANY valuations.

3) I came up with it! (Based on things already known.)

CON

CON

CON

CON

CON ϵ-Proportional, not proportional.

CON

CON ϵ-Proportional, not proportional. **CRUMBS!**.

CON

CON ϵ-Proportional, not proportional. **CRUMBS!**.

A and B need to be diamond cutters.

Is there a Proportional Super Cheat Proof Protocol?

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VOTE

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There is a no proportional super cheat proof protocol. The question is unknown to science.

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There is a no proportional super cheat proof protocol. The question is unknown to science.

The question was unknown to science until recently!

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There is a protocol to create 2 pieces, size $\frac{1}{2}$. Hence we can get super-cheat-proof.

What? The rest on the board.