

Introduction: Definitions of Fairness

William Gasarch-U of MD

Credit Where Credit is Due

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This is the main topic of this slide packet.

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Weird sets cause problems.

We will **ignore** those problems.

About Our Examples...

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All think they have $\geq \frac{1}{3}$.

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Then Ask Are A, B, C all happy with this?

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Are A, B, C happy with this allocation?

B and *C* are Green with Envy

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A's view: *A* gets 6. *B* gets 4. *C* gets 2.

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C's view: *A* gets 2. *B* gets 6. *C* gets 4.

A is happy a a claim. Are clams really happy?

B is envious of *A*.

B and *C* are Green with Envy

	l_1	l_2	l_3	l_4	l_5	l_6
<i>A</i>	2	2	2	2	2	2
<i>B</i>	1	1	2	2	3	3
<i>C</i>	6	4	1	1	0	0

A's view: *A* gets 6. *B* gets 4. *C* gets 2.

B's view: *A* gets 7. *B* gets 4. *C* gets 1.

C's view: *A* gets 2. *B* gets 6. *C* gets 4.

A is happy a a claim. Are clams really happy?

B is envious of *A*.

C is envious of *B*.

Envy-Free Allocations

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The allocation is **Envy-Free** if

Envy-Free Allocations

The allocation is **Envy-Free** if

$$V_A(P_A) \geq V_A(P_B), V_A(P_C).$$

Envy-Free Allocations

The allocation is **Envy-Free** if

$$V_A(P_A) \geq V_A(P_B), V_A(P_C).$$

$$V_B(P_B) \geq V_B(P_A), V_B(P_C).$$

Envy-Free Allocations

The allocation is **Envy-Free** if

$$V_A(P_A) \geq V_A(P_B), V_A(P_C).$$

$$V_B(P_B) \geq V_B(P_A), V_B(P_C).$$

$$V_C(P_C) \geq V_C(P_A), V_C(P_B).$$

Envy-Free Allocations

The allocation is **Envy-Free** if

$$V_A(P_A) \geq V_A(P_B), V_A(P_C).$$

$$V_B(P_B) \geq V_B(P_A), V_B(P_C).$$

$$V_C(P_C) \geq V_C(P_A), V_C(P_B).$$

All think they got the biggest piece (or tied).

Can You Give an Envy-Free Allocation

Can You Give an Envy-Free Allocation

Consider this valuation

Can You Give an Envy-Free Allocation

Consider this valuation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

Can You Give an Envy-Free Allocation

Consider this valuation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

Do With Neighbor Find an Envy-Free Allocation.

An Envy-Free Allocation

An Envy-Free Allocation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

An Envy-Free Allocation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4.

An Envy-Free Allocation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4. B gets 4.

An Envy-Free Allocation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

An Envy-Free Allocation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4.

An Envy-Free Allocation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4.

An Envy-Free Allocation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4. C gets 2.

An Envy-Free Allocation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4. C gets 2.

C's View: A gets 0.

An Envy-Free Allocation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4. C gets 2.

C's View: A gets 0. B gets 2.

An Envy-Free Allocation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4. C gets 2.

C's View: A gets 0. B gets 2. C gets 10.

An Envy-Free Allocation

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4. C gets 2.

C's View: A gets 0. B gets 2. C gets 10.

Nobody is envious. So is there still a reason to be unhappy?
Discuss.

What If They Find Out The Others Val?

What If They Find Out The Others Val?

	l_1	l_2	l_3	l_4	l_5	l_6
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B	1	1	2	2	3	3
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What If They Find Out The Others Val?

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
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C	6	4	1	1	0	0

A's View: A gets 4.

What If They Find Out The Others Val?

	l_1	l_2	l_3	l_4	l_5	l_6
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B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4. B gets 4.

What If They Find Out The Others Val?

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
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A's View: A gets 4. B gets 4. C gets 4.

What If They Find Out The Others Val?

	l_1	l_2	l_3	l_4	l_5	l_6
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A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4.

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	l_1	l_2	l_3	l_4	l_5	l_6
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A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4.

What If They Find Out The Others Val?

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4. C gets 2.

What If They Find Out The Others Val?

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
B	1	1	2	2	3	3
C	6	4	1	1	0	0

A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4. C gets 2.

C's View: A gets 0.

What If They Find Out The Others Val?

	l_1	l_2	l_3	l_4	l_5	l_6
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A's View: A gets 4. B gets 4. C gets 4.

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C's View: A gets 0. B gets 2.

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	l_1	l_2	l_3	l_4	l_5	l_6
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What If They Find Out The Others Val?

	l_1	l_2	l_3	l_4	l_5	l_6
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A's View: A gets 4. B gets 4. C gets 4.

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C's View: A gets 0. B gets 2. C gets 10.

Later A finds out C's valuation and realize that C got 10 in C's mind.

What If They Find Out The Others Val?

	l_1	l_2	l_3	l_4	l_5	l_6
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A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4. C gets 2.

C's View: A gets 0. B gets 2. C gets 10.

Later A finds out C's valuation and realize that C got 10 in C's mind.

So A is mad!

What If They Find Out The Others Val?

	l_1	l_2	l_3	l_4	l_5	l_6
A	2	2	2	2	2	2
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A's View: A gets 4. B gets 4. C gets 4.

B's View: A gets 4. B gets 4. C gets 2.

C's View: A gets 0. B gets 2. C gets 10.

Later A finds out C's valuation and realize that C got 10 in C's mind.

So A is mad!

Editorial Note I think A is being petty and stupid.