Discrete Proportional Protocols

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We will always show that if a player cheats (does not follow what they should do) then they may end up with **less** than if they didn't cheat.

Valuations For Cakes

Reminder

$V_A(Q)$ is how much Player A values piece Q.

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We show that if Alice cheats she could end up with less.

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3) Alice ends up with a piece that is $\frac{1}{4}$.
She deserves to get less.

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B and C do Cut and Choose on $(P1 \cup P2)$.

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4) There are scenarios where A envies B or C.

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Let $W(n)$ be the max number of cuts used.
In the worst case everyone trims so $W(n) = n - 1 + W(n-1)$.
$W(n) = n - 1 + W(n-1) = (n-1) + (n-2) + W(n-2) = \cdots (\sum_{i=2}^{n-1} i) + W(2) \sim n^2$.

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Note Alice thinks all 3 of her pieces are $\geq \frac{1}{6}$.
Note Bob thinks all 3 of his pieces are $\geq \frac{1}{6}$.

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4) B divides his piece in 3 (equally).
Note Alice thinks all 3 of her pieces are $\geq \frac{1}{6}$.
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5) Carol takes one of Alice's pieces and one of Bob's (biggest).

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Vote:

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Vote:
1) It is known that there is a protocol that uses $\leq n^{1.99}$ cuts (or some function $\ll n^2$).

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1) It is known that there is a protocol that uses $\leq n^{1.99}$ cuts (or some function $\ll n^2$).
2) It is known that every protocol uses around n^2 cuts.

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We will find out the number of cuts on the worksheet.

$n = 4$ Case of Divide and Conquer

Players are A, B, C, D.

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We assume:
$---A------B------C----------$
L is cake to the LEFT of B.
R is cake to the RIGHT of B.

$n = 4$ Case of Divide and Conquer

Players are A, B, C, D.

1) A, B, and C all simultaneously cut the cake $(\frac{1}{2}, \frac{1}{2})$

We assume:

$---A------B------C----------$

L is cake to the LEFT of B.

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Note that the protocol uses 5 cuts.

$n = 5$ Case of Divide and Conquer

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P could be the original pie or a piece of it or a set of pieces.

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Similar. Left to the reader.

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$C(2) = 1$

$C(3) = 3$

If $n \geq 4$ and n is even then $C(n) = 2C(\frac{n}{2}) + n - 1$.

If $n \geq 5$ and n is odd then
$C(n) = C((n-1)/2) + C((n+1)/2) + n - 1$.

On the worksheet we will get a rough upper bound on $C(n)$.