WORKSHEET ON DEFINITIONS OF FAIRNESS
SOLUTIONS

PROBLEM ONE:

	I_1	I_2	I_3	I_4	I_5	I_6
A						
B						
C						

Fill in the above with natural numbers such that the following hold.

- The numbers in a row add up to 12.
- There is an allocation where everyone gets 12 and nobody is envious.

PROBLEM TWO:

	I_1	I_2	I_3	I_4	I_5	I_6
A						
B						
C						

Fill in the above with natural numbers such that the following hold.

- The numbers in a row add up to 12.
- There is an allocation where
  - Everyone gets ≥ 4 (so its proportional).
  - A envies B
  - B envies C
  - C envies A
  - There is no other envy.

	I_1	I_2	I_3	I_4	I_5	I_6
A	**2**	**2**	5	5	0	0
B	0	0	**2**	**2**	5	5
C	5	5	0	0	**2**	**2**

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PROBLEM THREE

Show that there is a valuation and allocation for 10 people $A_1, \ldots, A_{10}$ such that

- YOU get to pick the number of items and the row-totals.
- The allocation is proportional.
- A_1 envies $A_2 B$
- A_2 envies A_3
- $\vdots$
- A_9 envies A_{10}
- A_{10} envies A_{11}
- There is no other envy.

	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}
A_1	**S**	L								
A_2		**S**	L							
A_3			**S**	L						
A_4				**S**	L					
A_5					**S**	L				
A_6						**S**	L			
A_7							**S**	L		
A_8								**S**	L	
A_9									**S**	L
A_{10}	L									**S**

Need that $\frac{S}{S+L} \geq \frac{S+L}{10}$ and $S < L$.
$10S \geq (S+L)^2$
We want S and L to be as far apart as possible.
If $S = 1$ then need $10 \geq (L+1)^2$ so $L = 2$
If $S = 2$ then need $20 \geq (L+2)^2$ so $L = 2$ DOES NOT WORK
If $S = 3$ then need $30 \geq (L+3)^2$ so $L = 2$ DOES NOT WORK
SO the ONLY values that work are $S = 1$, $L = 2$.

PROBLEM FOUR

Show that there is a valuation and allocation for 10 people $A_1, \ldots, A_{10}$ such that

- YOU get to pick the number of items and the row-totals.
- The allocation is proportional.
- Everyone envies everyone.
- You DO NOT have to use natural numbers.

	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}
A_1	**S**	L	L	L	L	L	L	L	L	L
A_2	L	**S**	L	L	L	L	L	L	L	L
A_3	L	L	**S**	L	L	L	L	L	L	L
A_4	L	L	L	**S**	L	L	L	L	L	L
A_5	L	L	L	L	**S**	L	L	L	L	L
A_6	L	L	L	L	L	**S**	L	L	L	L
A_7	L	L	L	L	L	L	**S**	L	L	L
A_8	L	L	L	L	L	L	L	**S**	L	L
A_9	L	L	L	L	L	L	L	L	**S**	L
A_{10}	L	L	L	L	L	L	L	L	L	**S**

Need that $\frac{S}{S+9L} \geq \frac{S+9L}{10}$ and $S < L$.
$10S \geq (S+9L)^2$
ChatGPT told me
$0 < S < \frac{1}{10}$ and $S < L < \frac{\sqrt{10S}-S}{9}$ works.
Example: $S = \frac{1}{100}$ and $L = \frac{1}{50}$.