

Solutions to Worksheet on Discrete Envy Free Protocols

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PROBLEM ONE

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A, B, C, D, E are the players.

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- 3) The allocation is envy-free.

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A, B, C, D, E are the players. P is the pie.

Find a protocol FIRST-PLAYER-HAPPY where:

- 1) All but T of the pie is allocated.
- 2) 1st player gets $\geq X$ (You fill in the X)
- 3) The allocation is envy-free.
- 4) We do not know what B, C, D, E think of T .

First Attempt at Solution

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1) A cuts P into 6 pieces (all equal).

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- 1) A cuts P into 6 pieces (all equal).
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Trimming put aside.
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Trimming put aside.
 - 4) D trims ≤ 1 piece (creates tie for biggest)
- Can't Work** B, C, D might have trimmed all 6 pieces.

Second Attempt at Solution

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1) A cuts P into 7 pieces (all equal).

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- 2) B trims ≤ 3 pieces (create 4-way tie for biggest)

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Trimming put aside.

4) D trims ≤ 1 piece (creates tie for biggest)

5) E takes a piece. Went first so no envy.

DOES NOT WORK If E took untrimmed then all the rest are trimmed so A will get trimmed piece.

Third Attempt at Solution

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1) A cuts P into 8 pieces (all equal).

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DOESN'T WORK Scenario:

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DOESN'T WORK Scenario:

B trims 3 pieces so have P'_1, P'_2, P'_3, P_4 tied for biggest.

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DOESN'T WORK Scenario:

B trims 3 pieces so have P'_1, P'_2, P'_3, P_4 tied for biggest.

C trims 2 P'_1 and P'_2 . So B only like P'_3 and P_4 .

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D trims 1 P'_3 . So B only likes P_4 .

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C trims 2 P'_1 and P'_2 . So B only like P'_3 and P_4 .

D trims 1 P'_3 . So B only likes P_4 .

E takes P_4 . B cannot get a piece he liked.

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 - 9) A takes a piece. Since 7 might be trimmed, and E might have taken trimmed, set $Y = 9$.
- A thinks his piece is $\frac{1}{9}$ so $X = \frac{1}{9}$.

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- 9) A takes a piece. Since 7 might be trimmed, and E might have taken trimmed, set $Y = 9$.

A thinks his piece is $\frac{1}{9}$ so $X = \frac{1}{9}$.

There are 2 pieces and trim left. E thinks that all $\leq \frac{6}{7}$.

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- 10) B takes a piece. Untrimmed piece left so no envy.

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- 9) C takes a piece. Must take piece she trimmed if available.
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- 10) B takes a piece. Untrimmed piece left so no envy.
- 11) A takes a piece. Untrimmed piece left so no envy.

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- 11) A takes a piece. Untrimmed piece left so no envy.

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There is a protocol ALL-PLAYER-HAPPY where:

- 1) All but T of the pie is allocated.
- 2) A, B, C, D all get $\geq \frac{1}{7}$ so they think T is $\leq \frac{6}{7}$.

PROBLEM THREE

A, B, C, D, E are the players. P is the pie.

There is a protocol ALL-PLAYER-HAPPY where:

- 1) All but T of the pie is allocated.
- 2) A, B, C, D all get $\geq \frac{1}{7}$ so they think T is $\leq \frac{6}{7}$.
- 3) The allocation is envy-free.

The All-Players-Happy Protocol

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1) FIRST-PLAYER-HAPPY($A, B, C, D, E; P$).

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The trim and piece set aside is S_1 .

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- 2) FIRST-PLAYER-HAPPY($B, C, D, E, A; S_1$).

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2) FIRST-PLAYER-HAPPY($B, C, D, E, A; S_1$).

The trim and piece set aside is S_2 .

The All-Players-Happy Protocol

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The trim and piece set aside is S_2 .

3) FIRST-PLAYER-HAPPY($C, D, E, A, B; S_2$).

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The trim and piece set aside is S_1 .

2) FIRST-PLAYER-HAPPY($B, C, D, E, A; S_1$).

The trim and piece set aside is S_2 .

3) FIRST-PLAYER-HAPPY($C, D, E, A, B; S_2$).

The trim and piece set aside is S_3 .

The All-Players-Happy Protocol

- 1) FIRST-PLAYER-HAPPY($A, B, C, D, E; P$).
The trim and piece set aside is S_1 .
- 2) FIRST-PLAYER-HAPPY($B, C, D, E, A; S_1$).
The trim and piece set aside is S_2 .
- 3) FIRST-PLAYER-HAPPY($C, D, E, A, B; S_2$).
The trim and piece set aside is S_3 .
- 4) FIRST-PLAYER-HAPPY($D, E, A, B, C; S_3$).

The All-Players-Happy Protocol

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The trim and piece set aside is S_2 .

3) FIRST-PLAYER-HAPPY($C, D, E, A, B; S_2$).

The trim and piece set aside is S_3 .

4) FIRST-PLAYER-HAPPY($D, E, A, B, C; S_3$).

One can show that all players think they have $\geq \frac{1}{7}$ so that what's left is $\leq \frac{6}{7}$.

PROBLEM FIVE-All-but ϵ Envy-Free

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there is a 5-person protocol All-but- ϵ Envy-Free where:

- 1) All but T of the pie is allocated.
- 2) The allocation is Envy-Free
- 3) All of the players think $T \leq \epsilon$.

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$$S_0 = P.$$

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For $i = 0$ to N

ALL-PLAYER-HAPPY($A, B, C, D, E; S_i$).

Let S_{i+1} be all that was set aside.

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For $i = 0$ to N

ALL-PLAYER-HAPPY($A, B, C, D, E; S_i$).

Let S_{i+1} be all that was set aside.

After N iterations all of the players think $S_N \leq (\frac{6}{7})^N$.

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ALL-PLAYER-HAPPY($A, B, C, D, E; S_i$).

Let S_{i+1} be all that was set aside.

After N iterations all of the players think $S_N \leq (\frac{6}{7})^N$.

For N large enough get $S_N \leq \epsilon$.