Solutions to Proportional Division Worksheet

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PROBLEM ONE

	I_1	I_2	I_3	I_4	I_5	I_6
A						
B						

Fill in this table such with natural numbers such that:

PROBLEM ONE

	l_1	l_2	l_3	l_4	l_5	l_6
A						
B						

Fill in this table such with natural numbers such that:
1) The numbers in a row add up to 12.

PROBLEM ONE

	I_1	I_2	I_3	I_4	I_5	I_6
A						
B						

Fill in this table such with natural numbers such that:
1) The numbers in a row add up to 12.
2) If A knows B's valuation then, in cut and choose, she can cut the cake so that she will do much better than 6 (which is half).

PROBLEM ONE SOLUTION

	I_1	I_2	I_3	I_4	I_5	I_6
A	1	1	3	3	2	2
B	3	4	1	1	1	2

PROBLEM ONE SOLUTION

	I_1	I_2	I_3	I_4	I_5	I_6
A	1	1	3	3	2	2
B	3	4	1	1	1	2

A can cut the cake $\{I_1, I_2\}$ $\{I_3, I_4, I_5, I_6\}$.

PROBLEM ONE SOLUTION

	I_1	I_2	I_3	I_4	I_5	I_6
A	1	1	3	3	2	2
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A can cut the cake $\{I_1, I_2\}$ $\{I_3, I_4, I_5, I_6\}$.

A know that B will pick
$\{I_1, I_2\}$ since it adds to 7.

PROBLEM ONE SOLUTION

	I_1	I_2	I_3	I_4	I_5	I_6
A	1	1	3	3	2	2
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A can cut the cake $\{I_1, I_2\}$ $\{I_3, I_4, I_5, I_6\}$.

A know that B will pick $\{I_1, I_2\}$ since it adds to 7.

A ends up with $\{I_3, I_4, I_5, I_6\}$ which adds up to 10.

PROBLEM TWO

	I_1	I_2	I_3	I_4	I_5	I_6
A						
B						
C						

Fill in this table such with natural numbers such that:

- The numbers in a row add up to 12.
- There is a way that if A cheats in the lone divider protocol, she will do worse than if she didn't.

PROBLEM TWO SOLUTION

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	2	2	3	2	1
B	3	3	3	3	0	0
C	2	2	1	1	3	3

PROBLEM TWO SOLUTION

	I_1	I_2	I_3	I_4	I_5	I_6
A	2	2	2	3	2	1
B	3	3	3	3	0	0
C	2	2	1	1	3	3

A cuts it $\{I_1, I_2, I_3, I_4\}$ (which is 9) and $\{I_5, I_6\}$ (which is 3).

PROBLEM TWO SOLUTION

	I_1	I_2	I_3	I_4	I_5	I_6
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C	2	2	1	1	3	3

A cuts it $\{I_1, I_2, I_3, I_4\}$ (which is 9) and $\{I_5, I_6\}$ (which is 3).
B takes $\{I_1, I_2, I_3, I_4\}$.

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C	2	2	1	1	3	3

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B takes $\{I_1, I_2, I_3, I_4\}$. A is left with $\{I_5, I_6\}$ which is 3.

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B takes $\{I_1, I_2, I_3, I_4\}$. A is left with $\{I_5, I_6\}$ which is 3.
Whatever happens later, A gets ≤ 3.

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A cuts it $\{I_1, I_2, I_3, I_4\}$ (which is 9) and $\{I_5, I_6\}$ (which is 3).
B takes $\{I_1, I_2, I_3, I_4\}$. A is left with $\{I_5, I_6\}$ which is 3.
Whatever happens later, A gets ≤ 3. Serves her right.

PROBLEM THREE

	I_1	I_2	I_3	I_4	I_5	I_6
A						
B						
C						

Fill in this table such with natural numbers such that:

- The numbers in a row add up to 12.
- There is a way that if A cheats in the Trim protocol, she will do worse than if she didn't.

PROBLEM THREE SOL

	I_1	I_2	I_3	I_4	I_5	I_6
A	1	2	2	2	2	3
B	2	2	2	2	2	2
C	2	2	2	2	2	2

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A cheats by cutting the piece $\{I_1, I_2\}$ which is worth $3 < 4$.

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A cheats by cutting the piece $\{I_1, I_2\}$ which is worth $3 < 4$. Perhaps he thinks someone will trim it so he ends up splitting $\{I_3, I_4, I_5, I_6\}$ which is worth 9 so he would end up with ≥ 4.5.

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A cheats by cutting the piece $\{I_1, I_2\}$ which is worth $3 < 4$. Perhaps he thinks someone will trim it so he ends up splitting $\{I_3, I_4, I_5, I_6\}$ which is worth 9 so he would end up with ≥ 4.5. B and C do not trim, so A is stuck with $\{I_1, I_2\}$.

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Generalize the Lone Chooser Algorithm to 4 players

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1) A, B, C do lone chooser so they each now have $\geq \frac{1}{3}$. Assmme A had P_A, B had P_B, C had P_C.

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1) A, B, C do lone chooser so they each now have $\geq \frac{1}{3}$. Assmme A had P_A, B had P_B, C had P_C.

2) A div P_A in 4 pieces; B div P_B in 4 pieces; C div P_C in 4 pieces (equal).

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A, B, C all have $\geq \frac{1}{3} - \frac{1}{4}\frac{1}{3} = \frac{1}{4}$.

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2) A div P_A in 4 pieces; B div P_B in 4 pieces; C div P_C in 4 pieces (equal).

3) D takes one piece from each of A, B, C.
A, B, C all have $\geq \frac{1}{3} - \frac{1}{4}\frac{1}{3} = \frac{1}{4}$.
D thinks P_A size x_A, P_B size x_B, P_A size x_C
where $x_A + x_B + x_C = 1$.

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3) D takes one piece from each of A, B, C.
A, B, C all have $\geq \frac{1}{3} - \frac{1}{4}\frac{1}{3} = \frac{1}{4}$.
D thinks P_A size x_A, P_B size x_B, P_A size x_C
where $x_A + x_B + x_C = 1$.
D has $\geq \frac{1}{4}x_A + \frac{1}{4}x_B + \frac{1}{4}x_C = \frac{1}{4}(x_A + x_B + x_C) = \frac{1}{4}$.

PROBLEM FIVE

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Do the last case of the Divide and Conquer Algorithm that I omitted.

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$C(2^m) = 2^i C(2^{m-i}) + i \times 2^m$

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Let $i = m - 1$ to get

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$C(2^m) = 2^iC(2^{m-i}) + i \times 2^m$
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$C(2^m) = 2^mC(2) + (m-1) \times 2^m = 2^m + (m-1) \times 2^m = m2^m$.

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$C(2^m) = 2(2C(2^{m-2}) + 2^{m-1}) + 2^m = 2^2 C(2^{m-2}) + 2 \times 2^m$
$C(2^m) = 2^i C(2^{m-i}) + i \times 2^m$
Let $i = m - 1$ to get
$C(2^m) = 2^m C(2) + (m-1) \times 2^m = 2^m + (m-1) \times 2^m = m2^m$.
If $n = 2^m$ then $m = \log_2 n$ so $T(n) = n \log n$.