WORKSHEET ON UNEQUAL DIVISION

PROBLEM ONE-Log Number of Cuts

Let $C(a, b)$ be the number of cuts needed for a protocol that ends up with A having $\geq \frac{a}{a+b}$ of the cake, and B having $\geq \frac{b}{a+b}$ of the cake.

Prove rigorously that $C(a, b) \leq \lceil \log_2(a + b) \rceil$.

PROBLEM TWO: A bit less than log cuts

Find several a, b, relatively primes, such that

$$C(a, b) \leq \lceil \log_2(a + b) \rceil - 2.$$

Try to show there are an infinite number of such examples.

PROBLEM TWO- The More the Merrier

A, B, and C want to cut a cake in ratio $a : b : c$.

Even though we have not shown this is possible yet (it is) let $C(a, b)$ be the number of cuts needed for a protocol that ends up with A having $\geq \frac{a}{a+b+c}$ of the cake, B having $\geq \frac{b}{a+b+c}$ of the cake, and C having $\geq \frac{c}{a+b+c}$ of the cake.

1) A, B, C want to cut a cake in ratio $5 : 8 : 11$. Give a protocol for this. This gives a bound on $C(5, 8, 11)$. Do you think your bound is optimal?

2) A, B, and C want to cut a cake in ratio $a : b : c$. Give a protocol for this. Roughly how many cuts does it use.

3) A_1, A_2, ..., A_t want to cut a cake in ratio $a_1 : \cdots : a_t$. Give a protocol for this. Roughly how many cuts does it use.