

The BEE Sequence

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See next page.

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Emily Kaplitz did the programming.

Erik Metz did the hard math.

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Mod 2

The First Few Values Mod 2

All $\equiv$ in this section are mod 2.

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{2}$

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Question $(\exists^\infty n)[a_n \equiv 0]$?

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Vote YES, NO, UNKNOWN TO BEE?

Lets Try To Spot a Pattern!

First some empirical observations.

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What do you notice about n , a_n and **Mod 2**? Discuss.

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If $n \equiv 1$ then $a_n \equiv 1$.

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If $n \equiv 1$ then $a_n \equiv 1$. Lets Prove This!

$n \equiv 1 \rightarrow a_n \equiv 1$. Induction

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$a_{2n+1} = a_{2n-1} + 2a_n \equiv a_{2n-1}$ by algebra and $2 \equiv 0 \pmod{2}$.

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$a_{2n-1} \equiv 1$ by the IH.

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Hence $a_{2n+1} \equiv a_{2n-1} \equiv 1$.

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Since for all ODD m , $a_m \equiv 1$ we have

$$a_{2n} = a_{2n-1} + a_n \equiv 1 + a_n.$$

$$(\exists^\infty n)[a_n \equiv 0]$$

This one does not need induction.

Since for all ODD m , $a_m \equiv 1$ we have

$$a_{2n} = a_{2n-1} + a_n \equiv 1 + a_n.$$

If n is odd then we have

$$a_{2n} = a_{2n-1} + a_n \equiv 1 + a_n \equiv 1 + 1 \equiv 0.$$

$$(\exists^\infty n)[a_n \equiv 0]$$

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Upshot For all k

$$a_{2(2k+1)} = a_{2(2k+1)-1} + a_{2k+1} \equiv 1 + 1 \equiv 0.$$

A Proof that Does Not Need Induction

Thm For all n

$$a_{n+1} \equiv 0 \vee a_{2n+1} \equiv 0 \vee a_{2n+2} \equiv 0.$$

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Pf

Case 1 $a_{n+1} \equiv 0$. DONE.

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Thm For all n

$$a_{n+1} \equiv 0 \vee a_{2n+1} \equiv 0 \vee a_{2n+2} \equiv 0.$$

Pf

Case 1 $a_{n+1} \equiv 0$. DONE.

Case 2 $a_{2n+1} \equiv 0$. DONE.

A Proof that Does Not Need Induction

Thm For all n

$$a_{n+1} \equiv 0 \vee a_{2n+1} \equiv 0 \vee a_{2n+2} \equiv 0.$$

Pf

Case 1 $a_{n+1} \equiv 0$. DONE.

Case 2 $a_{2n+1} \equiv 0$. DONE.

Case 3 $a_{n+1} \equiv 1$ and $a_{2n+1} \equiv 1$.

A Proof that Does Not Need Induction

Thm For all n

$$a_{n+1} \equiv 0 \vee a_{2n+1} \equiv 0 \vee a_{2n+2} \equiv 0.$$

Pf

Case 1 $a_{n+1} \equiv 0$. DONE.

Case 2 $a_{2n+1} \equiv 0$. DONE.

Case 3 $a_{n+1} \equiv 1$ and $a_{2n+1} \equiv 1$.

$$a_{2n+2} = a_{2n+1} + a_{n+1} \equiv 1 + 1 \equiv 0.$$

Which Proof Did You Like Better?

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1. The proof where we first show $a_m \equiv 1$ for odd m , and then show $a_{2(2k+1)} \equiv 0$.

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1. The proof where we first show $a_m \equiv 1$ for odd m , and then show $a_{2(2k+1)} \equiv 0$.
2. The proof where we showed that, for all n ,
 $a_{n+1} \equiv 0 \vee a_{2n+1} \equiv 0 \vee a_{2n+2} \equiv 0$.

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$\exists^\infty n$ with $a_n \equiv 0$

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$$a_{n+1} \equiv 0 \vee a_{2n+1} \equiv 0 \vee a_{2n+2} \equiv 0 \vee a_{2n+3} \equiv 0.$$

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Case 3a $a_{2n+1} \equiv 1$

$\exists^\infty n$ with $a_n \equiv 0$

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The BEE Sequence

Mod 4

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The BEE Sequence

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7	$a_7 = a_6 + a_3$	13	3
8	$a_8 = a_7 + a_4$	18	3
9	$a_9 = a_8 + a_4$	23	3
10	$a_{10} = a_9 + a_5$	30	0

The First Few Values Mod 5

In this section all $\equiv$ are mod 5.

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{5}$
1	a_1	1	1
2	$a_2 = a_1 + a_1$	2	2
3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	0
5	$a_5 = a_4 + a_2$	7	2
6	$a_6 = a_5 + a_3$	10	0
7	$a_7 = a_6 + a_3$	13	3
8	$a_8 = a_7 + a_4$	18	3
9	$a_9 = a_8 + a_4$	23	3
10	$a_{10} = a_9 + a_5$	30	0
11	$a_{11} = a_{10} + a_5$	37	2

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4	$a_4 = a_3 + a_2$	5	0
5	$a_5 = a_4 + a_2$	7	2
6	$a_6 = a_5 + a_3$	10	0
7	$a_7 = a_6 + a_3$	13	3
8	$a_8 = a_7 + a_4$	18	3
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No pattern here. But $a_4 \equiv a_6 \equiv 0$.

Lets Use $a_6 \equiv 0$

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Lets use that!

Lets Use $a_{11} \equiv a_{12} \equiv a_{13}$

Let $a_{11} \equiv a_{12} \equiv a_{13} \equiv r$

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Lets Use $a_{11} \equiv a_{12} \equiv a_{13}$

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Continued Next Slide.

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Case 1 $a_{21} \equiv 0$. DONE. Later cases assume $a_{21} \not\equiv 0$.

Case 2 Whats left.

One of $a_{21} + r$, $a_{21} + 2r$, $a_{21} + 3r$, $a_{21} + 4r$ is $\equiv 0$.

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We prove $(\exists^\infty n)[a_n \equiv 0]$.

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Continued on Next Slide.

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Lets use that!

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$$a_{4m-2} = a_{4m-3} + a_{2m-1} \equiv a_{4m-3} + r$$

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Lets Use $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}$

$$a_{4m-2} = a_{4m-3} + a_{2m-1} \equiv a_{4m-3} + r$$

$$a_{4m-1} = a_{4m-2} + a_{2m-1} \equiv a_{4m-3} + 2r$$

$$a_{4m} = a_{4m-1} + a_{2m} \equiv a_{4m-3} + 3r$$

$$a_{4m+1} = a_{4m} + a_{2m} \equiv a_{4m-3} + 4r$$

Case 0 $r \equiv 0$. DONE, $a_{2m-1} \equiv 0$. Later cases assume $r \not\equiv 0$.

Case 1 $a_{4m-3} \equiv 0$. DONE. Later cases assume $a_{4m-3} \not\equiv 0$.

Case 2 Whats left.

One of $a_{4m-3} + r$, $a_{4m-3} + 2r$, $a_{4m-3} + 3r$, $a_{4m-3} + 4r$ is $\equiv 0$.

Lets Use $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}$

$$a_{4m-2} = a_{4m-3} + a_{2m-1} \equiv a_{4m-3} + r$$

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Case 2 Whats left.

One of $a_{4m-3} + r$, $a_{4m-3} + 2r$, $a_{4m-3} + 3r$, $a_{4m-3} + 4r$ is $\equiv 0$.

So we have an $m' > m$ such that $a_{m'} \equiv 0$.

The BEE Sequence Mod 7

The First Few Values Mod 7

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In this section all $\equiv$ are mod 7

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{7}$

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n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{7}$
1	a_1	1	1

The First Few Values Mod 7

In this section all $\equiv$ are mod 7

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{7}$
1	a_1	1	1
2	$a_2 = a_1 + a_1$	2	2

The First Few Values Mod 7

In this section all $\equiv$ are mod 7

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{7}$
1	a_1	1	1
2	$a_2 = a_1 + a_1$	2	2
3	$a_3 = a_2 + a_1$	3	3

The First Few Values Mod 7

In this section all $\equiv$ are mod 7

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{7}$
1	a_1	1	1
2	$a_2 = a_1 + a_1$	2	2
3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5

The First Few Values Mod 7

In this section all $\equiv$ are mod 7

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{7}$
1	a_1	1	1
2	$a_2 = a_1 + a_1$	2	2
3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	0

The First Few Values Mod 7

In this section all $\equiv$ are mod 7

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{7}$
1	a_1	1	1
2	$a_2 = a_1 + a_1$	2	2
3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	0
6	$a_6 = a_5 + a_3$	10	3

The First Few Values Mod 7

In this section all $\equiv$ are mod 7

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{7}$
1	a_1	1	1
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3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	0
6	$a_6 = a_5 + a_3$	10	3
7	$a_7 = a_6 + a_3$	13	6

The First Few Values Mod 7

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3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	0
6	$a_6 = a_5 + a_3$	10	3
7	$a_7 = a_6 + a_3$	13	6
8	$a_8 = a_7 + a_4$	18	4

The First Few Values Mod 7

In this section all $\equiv$ are mod 7

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{7}$
1	a_1	1	1
2	$a_2 = a_1 + a_1$	2	2
3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	0
6	$a_6 = a_5 + a_3$	10	3
7	$a_7 = a_6 + a_3$	13	6
8	$a_8 = a_7 + a_4$	18	4
9	$a_9 = a_8 + a_4$	23	2

The First Few Values Mod 7

In this section all $\equiv$ are mod 7

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{7}$
1	a_1	1	1
2	$a_2 = a_1 + a_1$	2	2
3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	0
6	$a_6 = a_5 + a_3$	10	3
7	$a_7 = a_6 + a_3$	13	6
8	$a_8 = a_7 + a_4$	18	4
9	$a_9 = a_8 + a_4$	23	2
10	$a_{10} = a_9 + a_5$	30	2

The First Few Values Mod 7

In this section all $\equiv$ are mod 7

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{7}$
1	a_1	1	1
2	$a_2 = a_1 + a_1$	2	2
3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	0
6	$a_6 = a_5 + a_3$	10	3
7	$a_7 = a_6 + a_3$	13	6
8	$a_8 = a_7 + a_4$	18	4
9	$a_9 = a_8 + a_4$	23	2
10	$a_{10} = a_9 + a_5$	30	2
11	$a_{11} = a_{10} + a_5$	37	2

No pattern here. But $a_5 \equiv 0 \pmod{7}$.

Lets Try Same Approach as Mod 5

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We will use $a_m \equiv 0$ to get some larger m' with $a_{m'} \equiv 0$.

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$$a_{2m} = a_{2m-1} + a_m \equiv a_{2m-1}$$

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$$a_{2m} = a_{2m-1} + a_m \equiv a_{2m-1}$$

$$a_{2m+1} = a_{2m} + a_m \equiv a_{2m-1}$$

Lets Try Same Approach as Mod 5

We will use $a_m \equiv 0$ to get some larger m' with $a_{m'} \equiv 0$.

a_m is used for both a_{2m} and a_{2m+1} .

$$a_{2m} = a_{2m-1} + a_m \equiv a_{2m-1}$$

$$a_{2m+1} = a_{2m} + a_m \equiv a_{2m-1}$$

So we get

$$a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}.$$

Lets Try Same Approach as Mod 5

We will use $a_m \equiv 0$ to get some larger m' with $a_{m'} \equiv 0$.

a_m is used for both a_{2m} and a_{2m+1} .

$$a_{2m} = a_{2m-1} + a_m \equiv a_{2m-1}$$

$$a_{2m+1} = a_{2m} + a_m \equiv a_{2m-1}$$

So we get

$$a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}.$$

Lets use that!

Lets Use $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}$

WORK ON THIS IN GROUPS.

Lets Use $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}$

Lets Use $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}$

Let $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1} \equiv r$

Lets Use $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}$

Let $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1} \equiv r$

$$a_{4m-2} = a_{4m-3} + a_{2m-1} \equiv a_{4m-3} + r$$

Lets Use $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}$

Let $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1} \equiv r$

$$a_{4m-2} = a_{4m-3} + a_{2m-1} \equiv a_{4m-3} + r$$

$$a_{4m-1} = a_{4m-2} + a_{2m-1} \equiv a_{4m-3} + 2r$$

Lets Use $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}$

Let $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1} \equiv r$

$$a_{4m-2} = a_{4m-3} + a_{2m-1} \equiv a_{4m-3} + r$$

$$a_{4m-1} = a_{4m-2} + a_{2m-1} \equiv a_{4m-3} + 2r$$

$$a_{4m} = a_{4m-1} + a_{2m} \equiv a_{4m-3} + 3r$$

Lets Use $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}$

Let $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1} \equiv r$

$$a_{4m-2} = a_{4m-3} + a_{2m-1} \equiv a_{4m-3} + r$$

$$a_{4m-1} = a_{4m-2} + a_{2m-1} \equiv a_{4m-3} + 2r$$

$$a_{4m} = a_{4m-1} + a_{2m} \equiv a_{4m-3} + 3r$$

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Let $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1} \equiv r$

$$a_{4m-2} = a_{4m-3} + a_{2m-1} \equiv a_{4m-3} + r$$

$$a_{4m-1} = a_{4m-2} + a_{2m-1} \equiv a_{4m-3} + 2r$$

$$a_{4m} = a_{4m-1} + a_{2m} \equiv a_{4m-3} + 3r$$

$$a_{4m+1} = a_{4m} + a_{2m} \equiv a_{4m-3} + 4r$$

$$a_{4m+2} = a_{4m+1} + a_{2m+1} \equiv a_{4m-3} + 5r$$

Lets Use $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}$

Let $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1} \equiv r$

$$a_{4m-2} = a_{4m-3} + a_{2m-1} \equiv a_{4m-3} + r$$

$$a_{4m-1} = a_{4m-2} + a_{2m-1} \equiv a_{4m-3} + 2r$$

$$a_{4m} = a_{4m-1} + a_{2m} \equiv a_{4m-3} + 3r$$

$$a_{4m+1} = a_{4m} + a_{2m} \equiv a_{4m-3} + 4r$$

$$a_{4m+2} = a_{4m+1} + a_{2m+1} \equiv a_{4m-3} + 5r$$

$$a_{4m+3} = a_{4m+2} + a_{2m+1} \equiv a_{4m-3} + 6r$$

Lets Use $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}$

Let $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1} \equiv r$

$$a_{4m-2} = a_{4m-3} + a_{2m-1} \equiv a_{4m-3} + r$$

$$a_{4m-1} = a_{4m-2} + a_{2m-1} \equiv a_{4m-3} + 2r$$

$$a_{4m} = a_{4m-1} + a_{2m} \equiv a_{4m-3} + 3r$$

$$a_{4m+1} = a_{4m} + a_{2m} \equiv a_{4m-3} + 4r$$

$$a_{4m+2} = a_{4m+1} + a_{2m+1} \equiv a_{4m-3} + 5r$$

$$a_{4m+3} = a_{4m+2} + a_{2m+1} \equiv a_{4m-3} + 6r$$

Since have $\{r, 2r, 3r, 4r, 5r, 6r\}$ proof is similar to Mod 5.

The BEE Sequence Mod 9

The First Few Values Mod 9

(We skip mod 8 since mod 4 didn't work).

In this section all $\equiv$ are mod 9.

n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{9}$

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1	a_1	1	1

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1	a_1	1	1
2	$a_2 = a_1 + a_1$	2	2

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1	a_1	1	1
2	$a_2 = a_1 + a_1$	2	2
3	$a_3 = a_2 + a_1$	3	3

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1	a_1	1	1
2	$a_2 = a_1 + a_1$	2	2
3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5

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3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	7

The First Few Values Mod 9

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n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{9}$
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2	$a_2 = a_1 + a_1$	2	2
3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	7
6	$a_6 = a_5 + a_3$	10	1

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n	$a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}$	a_n	$a_n \pmod{9}$
1	a_1	1	1
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3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	7
6	$a_6 = a_5 + a_3$	10	1
7	$a_7 = a_6 + a_3$	13	4

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3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	7
6	$a_6 = a_5 + a_3$	10	1
7	$a_7 = a_6 + a_3$	13	4
8	$a_8 = a_7 + a_4$	18	0

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4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	7
6	$a_6 = a_5 + a_3$	10	1
7	$a_7 = a_6 + a_3$	13	4
8	$a_8 = a_7 + a_4$	18	0
9	$a_9 = a_8 + a_4$	23	5

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In this section all $\equiv$ are mod 9.

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4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	7
6	$a_6 = a_5 + a_3$	10	1
7	$a_7 = a_6 + a_3$	13	4
8	$a_8 = a_7 + a_4$	18	0
9	$a_9 = a_8 + a_4$	23	5
10	$a_{10} = a_9 + a_5$	30	3

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4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	7
6	$a_6 = a_5 + a_3$	10	1
7	$a_7 = a_6 + a_3$	13	4
8	$a_8 = a_7 + a_4$	18	0
9	$a_9 = a_8 + a_4$	23	5
10	$a_{10} = a_9 + a_5$	30	3
11	$a_{11} = a_{10} + a_5$	37	1

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3	$a_3 = a_2 + a_1$	3	3
4	$a_4 = a_3 + a_2$	5	5
5	$a_5 = a_4 + a_2$	7	7
6	$a_6 = a_5 + a_3$	10	1
7	$a_7 = a_6 + a_3$	13	4
8	$a_8 = a_7 + a_4$	18	0
9	$a_9 = a_8 + a_4$	23	5
10	$a_{10} = a_9 + a_5$	30	3
11	$a_{11} = a_{10} + a_5$	37	1

No pattern here. But $a_8 \equiv 0 \pmod{9}$.

Lets Try Same Approach as Mod 5

All $\equiv$ are mod 9

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All $\equiv$ are mod 9

We get the same equation:

$$a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}.$$

Lets Try Same Approach as Mod 5

All $\equiv$ are mod 9

We get the same equation:

$$a_{2m-1} \equiv a_{2m} \equiv a_{2m+1}.$$

WORK IN GROUPS TO GET SOME $a_{m'} \equiv 0$.

Vote on Mod 9

I suspect you did not succeed. **Vote**

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I suspect you did not succeed. **Vote**

1. $(\exists^\infty n)[a_n \equiv 0]$ and this has been proven (with a new technique I have not shown yet).

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1. $(\exists^\infty n)[a_n \equiv 0]$ and this has been proven (with a new technique I have not shown yet).
2. There is NOT an infinite number of a_n with $a_n \equiv 0$ and this has been proven.

Vote on Mod 9

I suspect you did not succeed. **Vote**

1. $(\exists^\infty n)[a_n \equiv 0]$ and this has been proven (with a new technique I have not shown yet).
2. There is NOT an infinite number of a_n with $a_n \equiv 0$ and this has been proven.
3. The question is **UNKNOWN TO BEE**