π Projects for π

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4	3	4	3.5	0.14	0.0001
6	3.1	3.8	3.4	0.10	0.001
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We will later use these numbers to find out how good an approx $A(n)$ is and to prove $\pi \neq \frac{22}{7}$.

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See next slide

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For π use what python says it is.

Write a program that generates the following table (the numbers I have in it are made up, but you'll see the idea) but rather than go out to 3 go out to whatever is reasonable for your computer.

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How many places of π does $\sqrt{6\sum_{i=1}^{n} \frac{1}{i^2}}$ give?

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Thm $1 + \frac{1}{2^8} + \frac{1}{3^8} + \cdots = \frac{\pi^8}{9450}$.

For each of these find an infinite summation for π and do what you did in the last slide.

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You have tables like this:

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Use these tables to conjecture a statement of the following form:
Using method BLAH, $A(n)$ gives $f(n)$ digits of accuracy for π
For each method BLAH fill in the $f(n)$.

Other Rationals that are Close to π

$\frac{22}{7} = 3.1428\ldots$. This is bigger than π.
$\frac{333}{106} = 3.141509\ldots$. This is smaller than π.
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For the inscribe-circumscribe method see what value of n you need to show that the fractions above are NOT π. You've already done that for $\frac{22}{7}$ where the answer is $n = 96$ (though check that slightly lower does not work, like $n = 94$.)

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Look up how these numbers were derived and write that up.

Other Ways to Approximate π

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LOOK UP on the web OTHER ways to approximate π and see if you can use them to show that the fractions on the last page are NOT π. Also look into if error bounds are known for these sums.

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Look at this website:
https://en.wikipedia.org/wiki/Liouville_number
For a proof that Liouville's number is Transcendental.