

Generalized Poker Hand Counts: Exact Formulas and Tie-Aware Rankings

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Abstract

In a standard 13-rank, four-suit deck, the usual five-card categories have a familiar frequency order. I derive exact counts for single-copy decks with r ranks and $2 \leq s \leq 4$ suits, where the nine conventional categories form a complete classification. At $(r, s) = (9, 4)$, the formulas give 480 flushes and 1,728 full houses. I also derive exact counts for specified six-card patterns. Using these formulas, I give tie-aware ranking maps on the finite integer grid $4 \leq r \leq 20$, $2 \leq s \leq 4$, and a separate full-house/flush comparison on a larger grid. A sampler checks three five-card deck sizes; it does not prove the formulas. The diagrams report exact comparisons at the displayed grid points, not a global description of ranking regions.

Code and data. The counting and sampling code associated with this study is available at <https://github.com/Ronisnotasianfr>. The figure-generation script and CSV files accompanying this manuscript document the exact integer-grid comparisons shown below.

1 Introduction

A standard deck has 13 ranks and 4 suits. For five-card hands, the usual categories, from least frequent to most frequent, are straight flush, four of a kind, full house, flush, straight, three of a kind, two pair, one pair, and high card. A royal flush is one of the straight flushes, not an additional category in the counts below. The order is familiar, but its frequencies depend on the deck composition.

Short Deck (also called Six-Plus Hold'em) illustrates the effect of removing ranks. It uses nine ranks and four suits. Some Short Deck rules rank a flush above a full house, unlike standard poker [7]. In the five-card model used here, the counts at $(r, s) = (9, 4)$ have that same rarity order: there are 480 flushes and 1,728 full houses. This comparison is about a uniformly chosen five-card subset. It is not a calculation of the best five-card hand from a seven-card Texas Hold'em deal, and it does not establish why a particular ruleset chose its ranking.

This paper asks how exact five-card hand frequencies change as a single-copy deck varies in ranks and suits, and which pairwise comparisons change direction. It also derives counts for selected six-card patterns, which require additional rank-multiplicity categories.

For five-card hands, the nine conventional categories form an exclusive classification when $2 \leq s \leq 4$: at most four cards can share a rank, so five of a kind cannot occur. Within this range I derive all nine counts. For the full-house/flush comparison alone, the two pattern counts remain well-defined for larger suit counts, so I also report that pairwise comparison on the integer grid $4 \leq r \leq 20$, $2 \leq s \leq 10$. The six-card table is restricted to at most four suits, where the same single-copy assumption excludes five and six of a kind.

After deriving the five-card counts, I compare full houses and flushes using the sign of their exact difference. A custom sampler draws one million five-card hands at each of $(13, 4)$, $(9, 4)$, and $(6, 4)$. The ranking maps show the exact weak order of the named categories at each integer grid point: equal counts share a rank, and zero-count categories are marked. The maps are restricted to suit counts for which the stated taxonomy is complete.

2 Related Work

Packel's book and Alspach's poker-computation resources provide background on standard poker counts and related calculations [6, 1, 2]. They motivate the counting approach here: specify a hand type, count choices of ranks and suits, and divide by the total number of hands when a probability is wanted.

Williamson fixes four suits and varies the number of ranks. He counts whether a seven-card holding contains each five-card type, an inclusive count, and separately studies how often each type is declared at showdown. In that model, the frequency ranking stabilizes only at 761 ranks [9]. Here I count exclusive five-card categories and vary both ranks and suits. The sample spaces and questions differ, so Williamson's results are related but not directly comparable to the counts below.

Emert and Umbach, and Gadbois, discuss inconsistencies that can arise in wild-card poker, where a player's choice of what a wild card represents can affect the hand type

declared [4, 5]. That issue is distinct from the fixed-deck model here: I count each physical hand once under a predetermined category definition and include no wild cards.

Because Short Deck formats can differ in their ranking rules, the cited overview is used only for general context [7]. The model below counts uniformly chosen subsets of a fixed deck; it does not model a particular game's best-hand selection rule.

3 Model and Definitions

A deck has one card for every pair (i, j) , with rank $i \in \{1, \dots, r\}$ and suit $j \in \{1, \dots, s\}$. There are $N = rs$ cards. A hand is an unordered subset of the deck; all $\binom{rs}{d}$ subsets of size d are treated as equally likely. Throughout the five-card formula section, r and s are integers, $r \geq 4$, and $2 \leq s \leq 4$. We use $\binom{n}{k} = 0$ when $n < k$ or $k < 0$.

Label the Ace-like rank 1 and the other ranks $2, \dots, r$ in increasing order. The Ace-low sequence is $1, 2, 3, 4, 5$; the Ace-high wrap uses ranks $\{r-3, r-2, r-1, r, 1\}$. For five cards, let $S_5(r)$ be the number of distinct straight rank sets. There are $r-4$ non-wrapping intervals when $r \geq 5$. At $r = 5$, the Ace-high set duplicates the only interval; for $r \geq 6$, it adds one distinct set. Thus

$$S_5(r) = \begin{cases} 0, & r < 5, \\ 1, & r = 5, \\ r-3, & r \geq 6. \end{cases}$$

The five-card hand categories are exclusive and are defined by exact rank multiplicities:

- **Straight flush (SF)**: five consecutive distinct ranks in one suit.
- **Four of a kind (4K)**: four cards of one rank and one card of a different rank.
- **Full house (FH)**: three cards of one rank and two of another.
- **Flush**: five distinct ranks in one suit, excluding straight flushes.
- **Straight**: five consecutive distinct ranks, excluding straight flushes.
- **Three of a kind (3K)**: multiplicities $3 + 1 + 1$.
- **Two pair (2P)**: multiplicities $2 + 2 + 1$.
- **One pair (OP)**: multiplicities $2 + 1 + 1 + 1$.
- **High card (HC)**: five distinct ranks that are neither a straight nor a flush.

Because there is at most one card of each rank-suit pair, a rank occurs at most s times. Thus when $s \leq 4$, no five-card hand contains five of a kind, and the listed multiplicity patterns together with the all-distinct patterns cover every hand. The categories SF, Flush, and Straight are made disjoint by removing straight flushes from the last two. A royal flush is included in SF.

4 Exact Five-Card Counts

The counts below are for unordered hands. The rank-multiplicity formulas choose ranks and then suits; the straight and flush formulas remove their shared straight-flush cases.

Theorem 4.1 (Five-card count formulas). *For integers $r \geq 4$ and $2 \leq s \leq 4$, the category counts are*

$$\begin{aligned}
\text{SF}(r, s) &= sS_5(r), \\
4\text{K}(r, s) &= r \binom{s}{4} (rs - 4), \\
\text{FH}(r, s) &= r(r-1) \binom{s}{3} \binom{s}{2}, \\
\text{Flush}(r, s) &= s \left[\binom{r}{5} - S_5(r) \right], \\
\text{Straight}(r, s) &= S_5(r)(s^5 - s), \\
3\text{K}(r, s) &= r \binom{s}{3} \binom{r-1}{2} s^2, \\
2\text{P}(r, s) &= \binom{r}{2} \binom{s}{2}^2 (r-2)s, \\
\text{OP}(r, s) &= r \binom{s}{2} \binom{r-1}{3} s^3, \\
\text{HC}(r, s) &= \binom{rs}{5} - (\text{SF} + 4\text{K} + \text{FH} + \text{Flush} + \text{Straight} + 3\text{K} + 2\text{P} + \text{OP}).
\end{aligned}$$

Proof. There are $S_5(r)$ straight rank sequences. Choosing one of the s suits gives $sS_5(r)$ straight flushes. For a straight with mixed suits, each sequence has s^5 suit assignments, of which s are all one suit; summing over sequences gives $S_5(r)(s^5 - s)$. A flush is a choice of one suit and five ranks, with the $S_5(r)$ straight-flush rank sets removed, giving $s(\binom{r}{5} - S_5(r))$.

For four of a kind, choose the rank, all four suits at that rank, and a kicker among the remaining $rs - 4$ cards. Since $s \leq 4$, the kicker cannot be another card of the same rank. For a full house, choose the triple rank and its three suits, then a different pair rank and its

two suits. This gives $r(r-1)\binom{s}{3}\binom{s}{2}$.

For 3K, choose its rank and three suits, two different singleton ranks among the remaining $r-1$, and one suit for each singleton. For 2P, choose two pair ranks, two suits at each, and a kicker rank and suit. For OP, choose its pair rank and suits, three other ranks, and one suit at each. These choices give the remaining displayed formulas. The rank-multiplicity patterns are mutually exclusive. A hand with a repeated rank cannot be a straight or flush, since both require five distinct ranks. Among all-distinct hands, straight flushes have been removed from both Straight and Flush. Therefore the eight non-HC categories are disjoint, and HC is exactly the complement. This establishes the formulas as a partition of the $\binom{rs}{5}$ hands. $\square$

Table 1: Exact five-card counts for the standard deck and Short Deck parameters.

Category	(13, 4)	(9, 4)
SF	40	24
4K	624	288
FH	3,744	1,728
Flush	5,108	480
Straight	10,200	6,120
3K	54,912	16,128
2P	123,552	36,288
OP	1,098,240	193,536
HC	1,302,540	122,400
Total	2,598,960	376,992
$\binom{rs}{5}$	2,598,960	376,992

The total row is a consistency check. Since HC is defined as the residual, the equality of the sum and $\binom{rs}{5}$ is not an independent validation of the other counts; the partition argument in the proof is what establishes completeness.

5 Exact Comparisons

For a fixed deck, all categories have the same denominator $\binom{rs}{5}$. Comparing their counts is therefore equivalent to comparing their probabilities. In this paper, “rarer” means “has a smaller exact count among the specified exclusive categories.”

5.1 Straight flush, four of a kind, and full house

Proposition 5.1. *For a four-suit deck with $r \geq 5$, the five-card counts satisfy*

$$\text{SF} < 4\text{K} < \text{FH}.$$

Proof. At $r = 5$, the exceptional straight-sequence count is $S_5(5) = 1$, so

$$\text{SF} = 4, \quad 4\text{K} = 4 \cdot 5 \cdot 4 = 80, \quad \text{FH} = 24 \cdot 5 \cdot 4 = 480.$$

The claimed inequalities hold. For $r \geq 6$, $S_5(r) = r - 3$, and the formulas give

$$\text{SF} = 4(r - 3), \quad 4\text{K} = 4r(r - 1), \quad \text{FH} = 24r(r - 1).$$

Since $r - 3 < r(r - 1)$ for $r \geq 6$, the first inequality holds; the second follows from $4 < 24$ and $r(r - 1) > 0$. $\square$

This proposition is limited to four suits, the only suit count in the stated model where all three categories are nonzero. For two or three suits, four of a kind is impossible; an ordering that lists zero-count categories as if they were distinct would be misleading.

5.2 Full house compared with flush

Define the exact count difference

$$\Delta_{\text{FH,FI}}(r, s) = r(r - 1) \binom{s}{3} \binom{s}{2} - s \left[\binom{r}{5} - S_5(r) \right].$$

For integer deck parameters where these categories are defined, $\Delta_{\text{FH,FI}} < 0$ means a full house is rarer, $\Delta_{\text{FH,FI}} > 0$ means a flush is rarer, and equality means the two counts tie. Thus the equality equation

$$r(r - 1) \binom{s}{3} \binom{s}{2} = s \left[\binom{r}{5} - S_5(r) \right] \tag{1}$$

is an exact algebraic comparison, with the piecewise definition of S_5 understood. Physical decks have integer r and s ; a real root of a polynomial extension need not be an actual deck or a tie at an integer point.

At $(9, 4)$,

$$\text{FH} = 9 \cdot 8 \binom{4}{3} \binom{4}{2} = 1,728, \quad \text{Flush} = 4 \left[\binom{9}{5} - 6 \right] = 480.$$

Hence $\Delta_{\text{FH,Fl}}(9, 4) = 1, 248 > 0$. At $(12, 4)$, $\text{FH} = 3, 168$ and $\text{Flush} = 3, 132$; at $(13, 4)$, $\text{FH} = 3, 744$ and $\text{Flush} = 5, 108$. Thus for four suits the rarer category changes between these two consecutive rank counts; neither deck is a tie.

Table 2: Selected exact full-house and flush counts. The $(13, 6)$ row compares these two patterns only; it is not a complete nine-category hand partition.

(r, s)	FH	Flush	Rarer category
$(13, 4)$	3,744	5,108	FH
$(9, 4)$	1,728	480	Flush
$(6, 4)$	720	12	Flush
$(12, 4)$	3,168	3,132	Flush
$(13, 6)$	46,800	7,662	Flush
$(10, 3)$	270	735	FH

5.3 Two further comparisons proved directly from the formulas

For $r \geq 5$ and $2 \leq s \leq 4$, straight flushes are less frequent than non-flush straights because

$$\frac{\text{Straight}}{\text{SF}} = \frac{s^5 - s}{s} = s^4 - 1 > 1.$$

Here $S_5(r) > 0$. For the same domain, the two-pair and one-pair formulas give

$$\frac{2\text{P}}{\text{OP}} = \frac{3(s-1)}{2s(r-3)} < 1,$$

so two pair is less frequent than one pair. The ratio follows by cancelling common factors in the two formulas; the final inequality holds because $r \geq 5$ and $s \leq 4$.

These are individual comparisons, not a classification of all $\binom{9}{2} = 36$ pairs. The maps below show exact weak orders at the displayed integer parameters, but do not establish a global classification.

6 Computation and Ranking Diagrams

6.1 Five-card Monte Carlo check

The Monte Carlo check uses a custom classifier on five-card subsets sampled uniformly without replacement. The deck is represented by its rs rank-suit pairs, and samples are drawn using Python's `random.sample`. For each of $(13, 4)$, $(9, 4)$, and $(6, 4)$, the procedure

uses one million trials; initialize the pseudorandom generator with seed 42 once, before sampling the decks in that order.

For a category with exact probability p , the standard error for its simulated proportion is $\sqrt{p(1-p)/n}$. Table 3 reports the analytical probabilities and the sampled proportions from the stated one-million-trial procedure.

These simulations check the formulas computationally at three deck sizes but do not prove them. At $(9, 4)$, the simulated high-card proportion differs from the exact probability by about 2.3 standard errors; rarer categories also have greater sampling uncertainty.

Table 3: Analytical and Monte Carlo probabilities for three five-card decks ($n = 1,000,000$ per deck; seed 42 initialized once before sampling the decks in the listed order). Differences are simulated minus analytical probabilities.

(r, s)	Type	Exact p	Simulated p	Difference	SE
(13, 4)	SF	0.0000154	0.000009	-0.000006	0.000004
	4K	0.0002401	0.000221	-0.000019	0.000015
	FH	0.001441	0.001459	+0.000018	0.000038
	Flush	0.001965	0.001999	+0.000034	0.000044
	Straight	0.003925	0.003976	+0.000051	0.000063
	3K	0.02113	0.02118	+0.00005	0.00014
	2P	0.04754	0.04766	+0.00012	0.00021
	OP	0.4226	0.4231	+0.0005	0.0005
	HC	0.5012	0.5005	-0.0007	0.0005
(9, 4)	SF	0.0000637	0.000072	+0.000008	0.000008
	4K	0.000764	0.000734	-0.000030	0.000028
	FH	0.00458	0.00452	-0.00006	0.00007
	Flush	0.001273	0.001248	-0.000025	0.000036
	Straight	0.01623	0.01628	+0.00005	0.00013
	3K	0.04278	0.04279	+0.00001	0.00020
	2P	0.09626	0.09604	-0.00022	0.00029
	OP	0.5134	0.5126	-0.0008	0.0005
	HC	0.3247	0.3257	+0.0010	0.0005
(6, 4)	SF	0.000282	0.000308	+0.000026	0.000017
	4K	0.002823	0.002809	-0.000014	0.000053
	FH	0.01694	0.01674	-0.00020	0.00013
	Flush	0.000282	0.000256	-0.000026	0.000017
	Straight	0.07199	0.07210	+0.00011	0.00026
	3K	0.09034	0.09030	-0.00004	0.00029
	2P	0.20328	0.20324	-0.00004	0.00040
	OP	0.54207	0.54254	+0.00047	0.00050
	HC	0.07199	0.07171	-0.00028	0.00026

The displayed proportions are rounded; small last-digit differences can result when sub-

tracting them. The standard errors are computed from the analytical p , not from the simulated proportion.

The sampler labels ranks $0, \dots, r - 1$. Rotate these labels so code rank 0 is paper rank 2, code rank 1 is paper rank 3, and code rank $r - 1$ is the Ace (paper rank 1). Under this relabeling, its extra rank set is the Ace-low sequence and its final ordinary run is the Ace-high wrap, so the sampler and the formulas use the same straight sets.

The included figure generator produces the exact ranking diagrams and CSV data from the formulas.

6.2 Full-house/flush grid

Figure 1 compares the exact full-house and flush counts on the integer grid $4 \leq r \leq 20$, $2 \leq s \leq 10$. Where both counts are positive, the diagram shows which count is smaller or whether they tie. Hatched gray cells mark parameter values where at least one category has count zero, so no strict frequency ordering is assigned. For $s > 4$, this figure compares only these two exact patterns and does not represent a complete classification of all hands. The equality condition is equation (1); the diagram records the sign at the displayed integer points.

6.3 Tie-aware ranking maps

At each integer deck size, the ranking maps sort the exact category counts from least to most frequent. Each panel represents one category; the cell number is its dense rank among positive category counts, with rank 1 least frequent. Equal counts share a rank, and a dagger ($\dagger$) marks a zero-count category. Figure 2 shows the five-card categories; Figure 3 shows the six-card patterns defined in this paper. The accompanying CSV files provide the exact counts and weak orders at every grid point.

Both ranking maps cover integer ranks $4 \leq r \leq 20$ and suit counts $2 \leq s \leq 4$, where the stated taxonomies are complete. When $s \geq 5$, five of a kind is possible in five-card hands and additional multiplicity patterns occur in six-card hands; a complete ranking in that range requires an expanded taxonomy and new formulas. The maps report exact orderings at each displayed integer point; they make no assertion beyond the grid. For example, at $(6, 4)$ the five-card counts include the ties $SF = \text{Flush} = 12$ and $\text{Straight} = HC = 3, 060$.

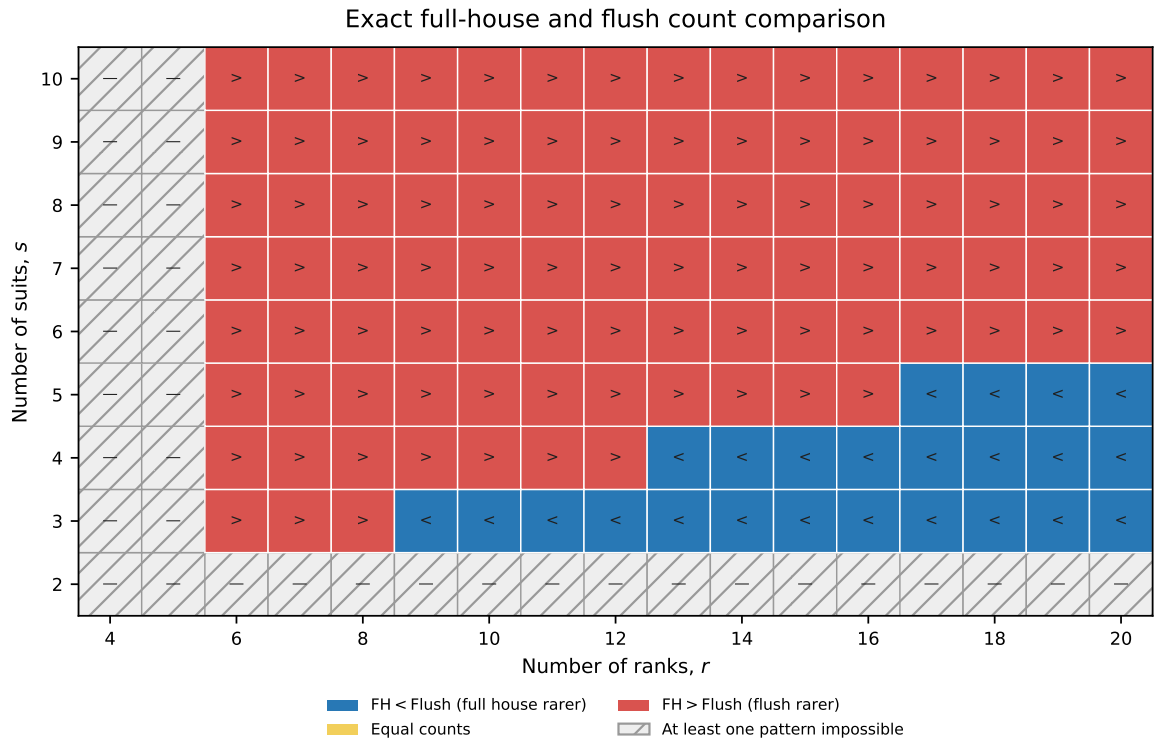


Figure 1: Exact full-house and flush count comparison on $4 \leq r \leq 20$, $2 \leq s \leq 10$. Blue indicates $\text{FH} < \text{Flush}$, gold indicates equal counts, and red indicates $\text{FH} > \text{Flush}$. Hatched gray cells mean at least one count is zero, so a strict comparison is not assigned. For $s > 4$, only these two patterns are compared.

Five-card category rankings on the integer grid

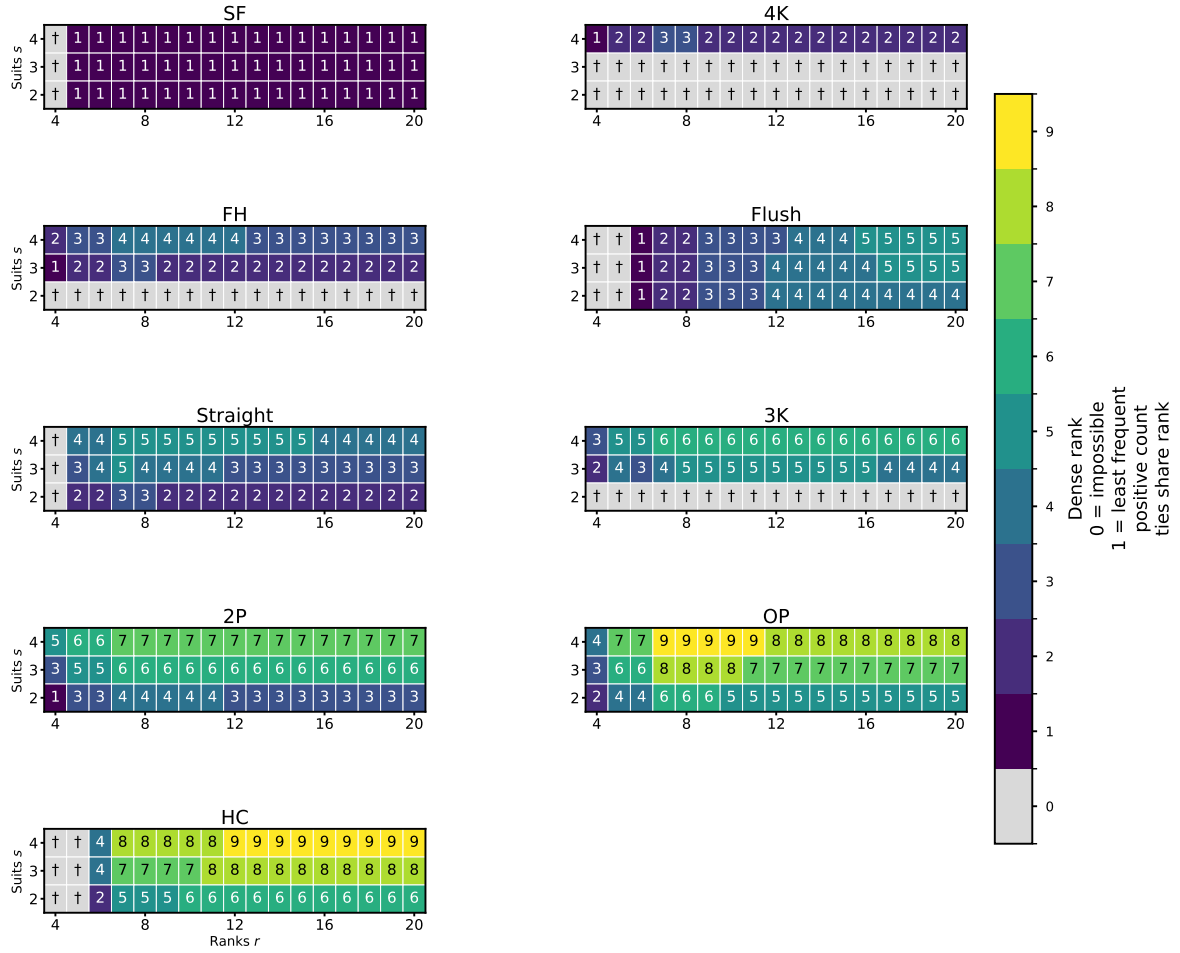


Figure 2: Dense frequency ranks of the nine five-card categories for $4 \leq r \leq 20$ and $2 \leq s \leq 4$. Rank 1 is least frequent; equal positive counts share a rank. A dagger marks a zero count (an impossible category). The exact counts and weak orders are in `five_card_rankings.csv`.

Six-card category rankings on the integer grid

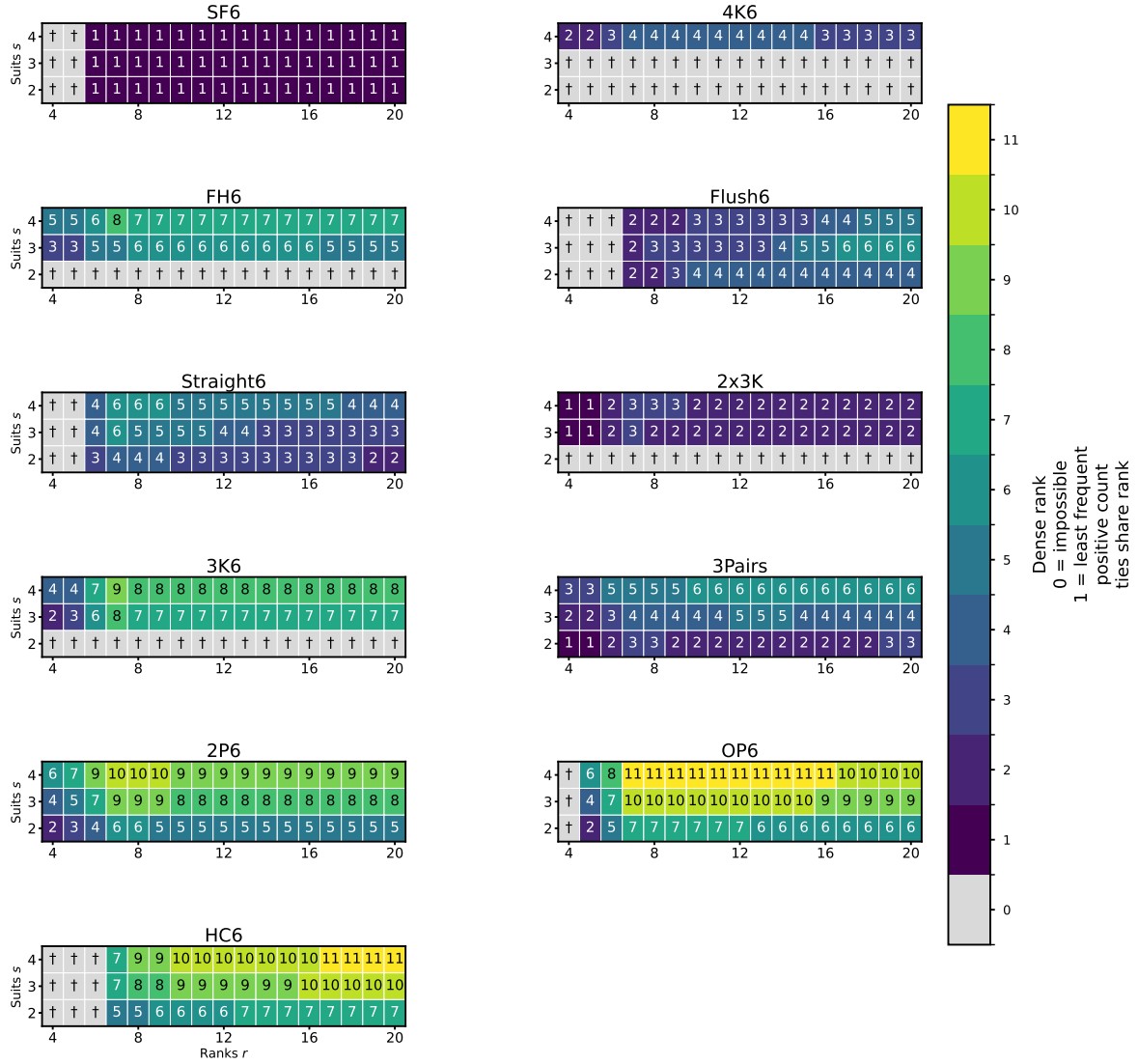


Figure 3: Dense frequency ranks of the eleven six-card patterns for $4 \leq r \leq 20$ and $2 \leq s \leq 4$, under the exact-pattern definitions in Section 7. Rank 1 is least frequent; equal positive counts share a rank. A dagger marks a zero count. The exact counts and weak orders are in `six_card_rankings.csv`.

7 Six-Card Extension

For six-card hands, I use the same single-copy deck and the same straight convention. The counts in this section are restricted to $2 \leq s \leq 4$; the numerical table uses $s = 4$. A six-card straight requires six distinct consecutive ranks, and a six-card flush requires six distinct ranks. I count $3 + 3$ as two three-of-a-kind and define a full house as $3 + 2 + 1$. A six-card four-of-a-kind category includes both $4 + 2$ and $4 + 1 + 1$. These are definitions for this count model, not a claim about every poker ruleset's best-five-card evaluation.

For six-card straights, the Ace-high wrap uses ranks $\{r - 4, r - 3, r - 2, r - 1, r, 1\}$. Let $S_6(r)$ be the number of distinct six-rank straight sets under this convention:

$$S_6(r) = \begin{cases} 0, & r < 6, \\ 1, & r = 6, \\ r - 4, & r \geq 7. \end{cases}$$

For $r \geq 7$, there are $r - 5$ non-wrapping intervals and one additional Ace-high set, giving $S_6(r) = r - 4$. At $r = 6$, the wrap and the only non-wrapping interval use the same six ranks, so $S_6(6) = 1$.

Proposition 7.1 (Six-card category counts for at most four suits). *For $r \geq 4$ and $2 \leq s \leq 4$,*

define categories by the exact patterns listed below. Their counts are

$$\begin{aligned}
\text{SF}_6 &= sS_6(r), \\
4\text{K}_6 &= r \binom{s}{4} \binom{rs-4}{2}, \\
\text{FH}_6 \ (3+2+1) &= r(r-1)(r-2) \binom{s}{3} \binom{s}{2} s, \\
\text{Flush}_6 &= s \left[\binom{r}{6} - S_6(r) \right], \\
\text{Straight}_6 &= S_6(r)(s^6 - s), \\
2 \times 3\text{K} \ (3+3) &= \binom{r}{2} \binom{s}{3}^2, \\
3\text{K}_6 \ (3+1+1+1) &= r \binom{s}{3} \binom{r-1}{3} s^3, \\
3\text{Pairs} \ (2+2+2) &= \binom{r}{3} \binom{s}{2}^3, \\
2\text{P}_6 \ (2+2+1+1) &= \binom{r}{2} \binom{s}{2}^2 \binom{r-2}{2} s^2, \\
\text{OP}_6 \ (2+1+1+1+1) &= r \binom{s}{2} \binom{r-1}{4} s^4, \\
\text{HC}_6 &= \binom{rs}{6} - (\text{the sum of the other ten counts}).
\end{aligned}$$

Proof. For the new multiplicity patterns, choose the ranks that carry each group and then their suits. Thus three pairs require three ranks and two suits at each; two triples require two ranks and three suits at each. For a six-card four of a kind, choose its rank and all four suits, then any two of the remaining cards; this includes the multiplicity patterns $4+2$ and $4+1+1$. Since $s \leq 4$, no rank can occur five or six times, and there is at most one rank with multiplicity four, so this choice counts each such hand once. The $3+2+1$, $3+1+1+1$, $2+2+1+1$, and $2+1+1+1+1$ formulas follow by choosing the corresponding distinct ranks and suits.

The six-card straight and flush arguments are the same as for five cards: there are $S_6(r)$ rank sets; each has s^6 suit assignments, of which s are all one suit. A flush chooses six ranks in one suit and omits the straight-flush rank sets. Every other six-card hand has a rank-multiplicity partition of 6 with maximum part at most 4. The displayed exact patterns cover those possibilities. The all-distinct remainder is split into straight flush, flush, straight, and high card, so the categories are disjoint and exhaustive. $\square$

Table 4: Six-card exact counts at four suits. These values use $S_6(13) = 9$ and $S_6(9) = 5$, including the distinct Ace-high wrap when $r \geq 7$.

Category	(13, 4)	(9, 4)
SF ₆	36	20
4K ₆	14,664	4,464
FH ₆ (3 + 2 + 1)	164,736	48,384
Flush ₆	6,828	316
Straight ₆	36,828	20,460
2 × 3K (3 + 3)	1,248	576
3K ₆ (3 + 1 + 1 + 1)	732,160	129,024
Three pairs (2 + 2 + 2)	61,776	18,144
2P ₆ (2 + 2 + 1 + 1)	2,471,040	435,456
OP ₆ (2 + 1 + 1 + 1 + 1)	9,884,160	967,680
HC ₆	6,985,044	323,268
Sum	20,358,520	1,947,792
$\binom{rs}{6}$	20,358,520	1,947,792

The high-card count is again a residual, so equality of the sum and $\binom{rs}{6}$ is an arithmetic consistency check, not an independent test of every formula. The accompanying figure generator implements the formulas in this section, including the additional Ace-high sequence when $r \geq 7$, and checks the category sums across the plotted grid. No six-card Monte Carlo trials are reported. For suit counts above four, additional multiplicity categories are needed for a complete six-card classification.

8 Possible Weak Orders

Nine labeled categories can be arranged into $\sum_{k=1}^9 S(9, k)k! = 7,087,261$ weak orders, where $S(n, k)$ is a Stirling number of the second kind. This is a count of all formal rankings with possible ties, not a count of poker rankings realized by deck parameters. The ordered Bell number formula appears in the combinatorics literature [3, 8].

9 Limitations and Next Questions

The exact five-card formulas are proved for the conventional nine-category model with $2 \leq s \leq 4$. At five or more suits, five of a kind is possible, so a complete ranking requires an expanded set of categories. The full-house and flush formulas still compare those two exact patterns, but they do not determine a complete hand hierarchy.

The five-card simulations cover only $(13, 4)$, $(9, 4)$, and $(6, 4)$; they provide sample-based checks, not proofs of the formulas. The six-card formulas are derived combinatorially and are not independently checked by simulation here. The ranking maps report tie-aware comparisons for $4 \leq r \leq 20$ and $2 \leq s \leq 4$. An analysis of best-five-of-seven Texas Hold'em hands would require a different sample space and a rule for selecting the best five-card category.

10 Conclusion

For the five-card model, direct rank-and-suit choices give the standard-deck counts and the Short Deck comparison exactly. At $(9, 4)$, a flush is rarer than a full house: there are 480 flushes and 1,728 full houses. Equating the two formulas gives an exact comparison condition; the diagram shows the sign of their difference at the displayed integer points.

The six-card formulas use exact rank-multiplicity patterns and at most four suits. With the Ace-high wrap included, a 13-rank deck has nine distinct six-rank sequences; the straight-flush, straight, flush, and residual high-card counts in Table 4 use this convention. No six-card Monte Carlo trials are reported here.

Extending the maps beyond the displayed grid, studying whether equal-order regions are connected, and analyzing best-five-of-seven Hold'em hands are natural directions for further work; each requires additional definitions and verification.

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