CMSC452 Elementary Theory of Computation, Fall 2001

Sketch of Solutions: Homework #1

Note: No partial credits are given to each graded unit.

Problem 1.1.1 [3+3+4=10 points]

(a) true. Every set is a subset of itself, including ∅. [3 points]

(e) true. {a,b} is a element of {a,b,c,{a,b}}. [3 points]

(g) false. Each element of a subset of a power set is a set, while a, b are not. [4 points]

Problem 1.1.2 [3+3+4=10 points]

(b) {3,5,7} [3 points]

(c) {1,2,7,9} [3 points]

(e) {∅}, or {{}} equivalently [4 points]

Problem 1.1.3 [5 points]

(d) If $a \in A$, then $A \in A \cup (A \cap B)$. If $a \notin A$, then $a \notin A \cap B$, thus $a \notin A \cup (A \cap B)$. Finally $A = A \cup (A \cap B)$.

Problem 1.1.4 [5+5=10 points]

(a) Fewest:{{a,b,c,d}} [5 points], Most:{{a},{b},{c},{d}} [5 points]

Problem 1.2.2 [3+3+4=10 points]

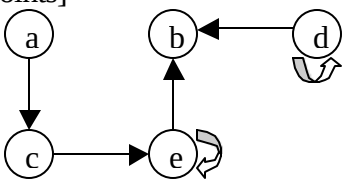
$R \circ R = \{(a,a),(a,d),(a,b),(a,c),(b,a),(b,b),(b,c)\}$ [3 points]

$R^{-1} = \{(b,a),(c,a),(d,c),(a,a),(a,b)\}$ [3 points]

R, $R \circ R$, R^{-1} are not functions. [4 points]

Problem 1.3.1 [5+5=10 points]

(a) [5 points]



(d) [5 points]

Problem 1.3.9 [5 points]

A directed graph represents a function when there is exactly one arraw/edge leading out of each node.

Problem 1.4.1(b) [10 points]

For a given natural number n, there are only finite many ways of adding up distanct natural numbers to make n. For example, five=5=1+4=2+3. Define N_n be the set of all sets of distinct natural numbers whose sum is n. Each N_n is finite and thus countable. Therefore, the set of union of all N_n, is countable.

(ps. N_n={∅,{1},{2},{3},{1,2},{4},{1,3},{5},{1,4},{2,3},..})

Problem 1.5.9 [10 points]

Suppose set S be uncountable and set T be countable. Suppose S-T were countable. Then S would be countable because it is the union of the countable sets S and S-T. Thus S-T cannot be countable.

Problem 1.7.6(b) [5 points]

$L^+ = L^* - \{e\}$ if and only if $e \notin L$.

Problem 1.8.2 [5 points]

(a) $(a \cup b)^*$

Problem 1.8.5 [5+5=10 points]

(a) true [5 points]

(c) false, $a^* b^* \cap b^* c^* = b^*$ [5 points]