CMSC452 Elementary Theory of Computation, Fall 2001

Sketch of Solutions: Homework 2

Note: 5 partial credits are given to each graded unit of problems 2.1.3, 2.1.5 and 2.2.9 for minor errors. No partial credits are given to the rest problems.

Problem 2.1.1 [5+5=10 points]

$e \in L(M)$ if and only if $s \in F$. [5 points]

<Proof:> Suppose $e \in L(M)$. Then $(s,e) \vdash^*_M (q,e)$ for some $q \in F$. But e is the empty string, so $(s,e)=(q,e)$. Therefore $s=q$ and thus $s \in F$. On the other hand, if $s \in F$, then by definition, $e \in L(M)$. [5 points]

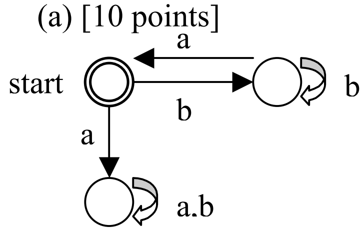
Problem 2.1.2 [5+5=10 points]

Top Left Machine: $a(ba)^*$ [5 points]

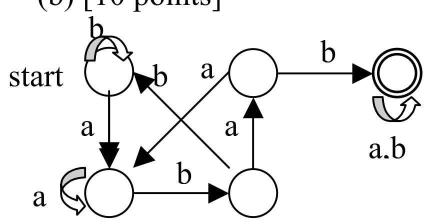
Middle Right Machine: $(ab \cup ba)^*$ [5 points]

Problem 2.1.3 [10+10=20 points]

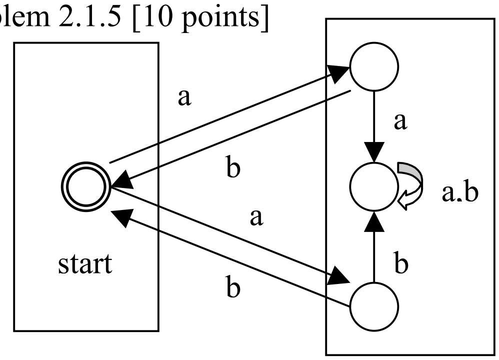
(a) [10 points]



(b) [10 points]



Problem 2.1.5 [10 points]



Problem 2.1.7 [10 points]

By definition, $(q_0,w) \vdash^*_M (q,e) \Rightarrow (q_0,w) \vdash_M (q_1,a_1w_1) \vdash_M (q_2,a_1\, a_2w_s) \vdash_M (q_3,a_1\, a_2\, a_3w_3) \vdash_M \ldots \vdash_M (q_n,e)$, where $q_1,q_2,\ldots,q_n$ are reachable states and $q_n=q$. Unreachable states will never appear in the list. Therefore, removing unreachable states from K doesn't change the language accepted. [5 points]

Define graph $G=(V,E)$ corresponding to a given deterministic finite automation $M=(K, \Sigma, \delta, q_0, F)$ by $V=K$, $(q_i, q_j) \in E$ if $(q_i,c) \vdash_M (q_j,e)$ for some $c \in \Sigma$. Run *DFS* (depth first search) with starting vertex q_0 to find all the reachable vertices. [5 points]

Problem 2.2.1 (b) [2×5=10 points]

e, ab,$abab$, aba are accepted. [2×4=8 points]

$abaa$ is rejected. [2 points]

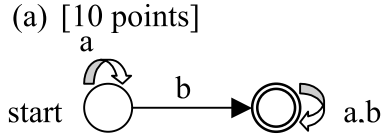
Problem 2.2.2 [5+5=10 points]

(a) a^*.

(b) $e\cup(a(ba\cup baa)^*(b\cup ba))$ or equivalently $(ab\cup aba)^*$.

Problem 2.2.9 [10+10=20 points]

(a) [10 points]



(b) [10 points]