CMSC452 Elementary Theory of Computation, Fall 2001

Sketch of Solutions: Homework 3

Note: 5 partial credits are given to each of problems 2.2.6(a)*,* 2,3,4(c)*,* 2.4.4*,* 2.3.5(a) *and* 2.4.10(b) *for minor errors. No partial credits are given to the rest problems.*

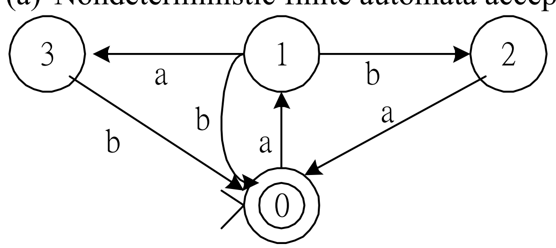
Problem 2.2.4 [5+5=10 points]

(a) Since $\triangle=\{(q_i,a_j,q_i):i\neq j\}$, once a machine is in state q_i, then it will never leave q_i. Moreover, the machine can only grant/take a_j for $j\neq i$. Thus if the machine starts with state q_i, it accepts strings missing at least the symbol a_i. And in this non-deterministic finite automaton, the machine can start from any state. So it accepts the language consisting of all the strings missing at least one symbol. [5 points]

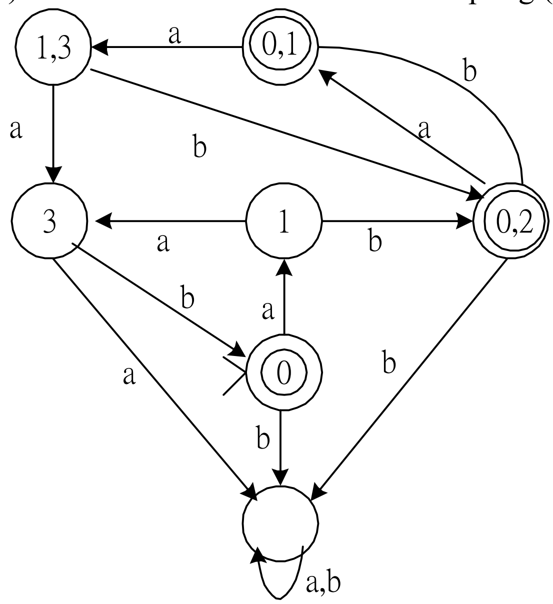
(b) Adding a new start state and transitions on e from it to each of the former states, and making all the previous start states no longer initial. Then the machine has only 1 start state as normal definition and accepts the same language. [5 points]

Problem 2.2.6 [10+5+5=20 points]

(a) Nondeterministic finite automata accepting $(ab\cup aab\cup aba)^*$ [10 points]



(b) Deterministic finite automata accepting $(ab\cup aab\cup aba)^*$ [5 points]



(c) No simpler deterministic machine accepting $(ab\cup aab\cup aba)^*$ exists. It can be verified by state-minimization algorithm in section 2.5. [5 points]

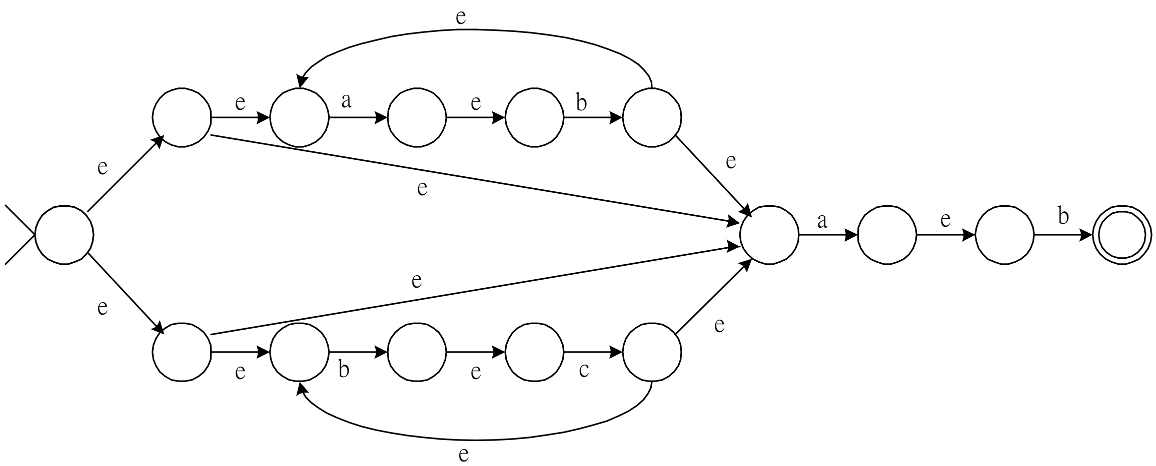
Problem 2.3.1 [5+5=10 points]

(a) If we interchange the final and non-final states of a nondeterministic finite automaton, then the strings it formerly accepted are not guaranteed to be rejected. [5 points]

(b) There are examples of nondeterministic finite automata such that $(s,w) \vdash^*_M (q_1,e)$ and $(s,w) \vdash^*_M (q_2,e)$ for some string w and final state q_1 and non-final state q_2. Then not only $w \in L(M)$ but also the automaton resulting from interchanging start and final states accept w. [5 points]

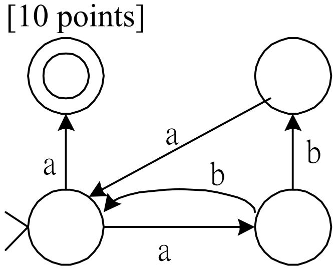
Problem 2.3.4 (c) [10 points]

$((ab)^* \cup (bc)^*)ab$

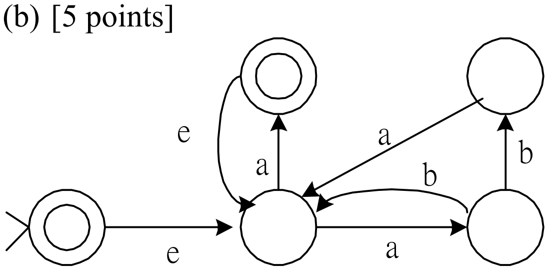


Problem 2.3.5 [10+5=15 points]

(a) [10 points]



(b) [5 points]



Problem 2.3.7 (d) [5 points]

$(a \cup ba^*a)(ba^*a)^*b(b \cup a)^*$

Problem 2.4.4 [10 points]

Suppose $m=n=k$, then $L=\{w=a^k ba^k ba^{2k}: k\geq 1\}$. Assume L is a regular expression. By theorem 2.4.1 (pumping lemma), there is an $N\geq 1$ such that if $w\in L$ and $|w|\geq N$, w can be written as $w=xyz$ where $y\neq e$, $|xy|\leq N$, and $xy^i z$ can be expressed as $a^k ba^k ba^{2k}$ for all $i\geq 0$. We know w contains 2 bs. Then y cannot contain any b; otherwise, $xy^0 z$ has no b and $xy^0 z\notin L$. Now there are 3 possibilities: (1) y is before the first b; (2) y is between the two bs; and (3) y is after the second b. For case (1), $xy^2 z= a^{k+|y|}ba^k ba^{2k}\notin L$ since $k+|y|\neq k$. Contradiction. Similarly, case (2) and (3) lead to contradiction. Thus, L is not a regular expression.

Problem 2.4.8 (a) [5+5=10 points]

False. [5 points]

Each language, regular or not regular, is a subset of the regular language Σ^*. [5 points]

Problem 2.4.10 (b) [10 points]

An example of deterministic 2-head finite automaton accepting wcw where $w=(a\cup b)^*$