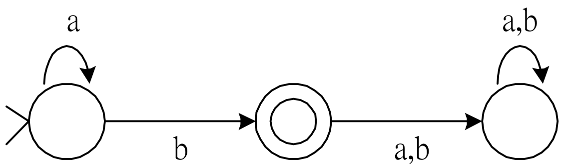
CMSC452 Elementary Theory of Computation, Fall 2001

Sketch of Solutions: Homework 4

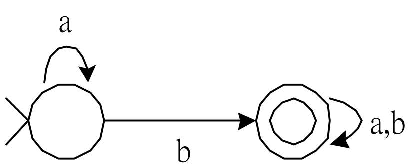
Note: 5 partial credits are given to each graded unit of problems 2.5.3 *and* 3.3.2(b) *for minor errors. No partial credits are given to the rest problems.*

Problem 2.5.3 [10+10=20 points]

Upper right machine of 2.1.2 [10 points]



Machine (a) of 2.2.9 [10 points]



Problem 2.6.1 (b) [5 points]

For example, bb can be generated by $aa(a\cup b)^* \cup (bb)^*a^*$ but cannot be generated by $(ab\cup ba\cup a)^*$.

Problem 3.1.1 [4×1+4×2+3=15 points]

(a) aa, baa, aba, aab. [4×1=4 points]

(b) $S\Rightarrow AA\Rightarrow bAA\Rightarrow bAAb\Rightarrow bAbAb\Rightarrow bAbbAb\Rightarrow babbAb\Rightarrow babbab$

$S\Rightarrow AA\Rightarrow bAA\Rightarrow bAAb\Rightarrow bAbAb\Rightarrow bAbbAb\Rightarrow bAbbab\Rightarrow babbab$

$S\Rightarrow AA\Rightarrow bAA\Rightarrow bAbA\Rightarrow babA\Rightarrow babbA\Rightarrow babbAb\Rightarrow babbab$

$S\Rightarrow AA\Rightarrow bAA\Rightarrow bAAb\Rightarrow baAb\Rightarrow babAb\Rightarrow babbAb\Rightarrow babbab$

[4×2=8 points]

(c) $S\Rightarrow AA\Rightarrow\ldots\Rightarrow b^mAA\Rightarrow\ldots\Rightarrow b^mAb^nA\Rightarrow\ldots\Rightarrow b^mAb^nAb^p\Rightarrow b^mAb^nab^p\Rightarrow b^mab^nab^p$

[3 points]

Problem 3.1.3 (b) [5 points]

Context-free grammar for $\{ww^R\text{: } w\in\{a,b\}^*\}$:

$G=\{V,\Sigma,R,S\}$ where $V=\{a,b,S\}$, $\Sigma=\{a,b\}$, $R=\{S\rightarrow aSa,\ S\rightarrow bSb,\ S\rightarrow e\}$

Problem 3.1.5 (a) [5 points]

$S\Rightarrow aB\Rightarrow abS\Rightarrow abaB\Rightarrow abaB\Rightarrow ababS\Rightarrow ababbA\Rightarrow ababba$

Problem 3.1.9 (f) [5 points]

Context-free grammar for $\{a^mb^n\text{: } m\le 2n\}$:

$G=\{V,\Sigma,R,S\}$ where $V=\{a,b,A,S\}$, $\Sigma=\{a,b\}$, $R=\{S\rightarrow AASb,\ S\rightarrow e,\ A\rightarrow a,\ A\rightarrow e\}$.

Problem 3.2.2 [5+5=10 points]

$S \Rightarrow (S) \Rightarrow ()$

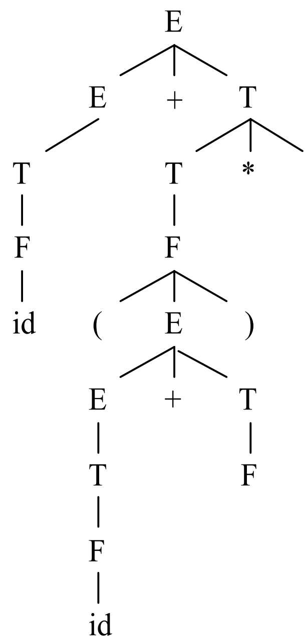
$S \Rightarrow SS \Rightarrow S(S) \Rightarrow S() \Rightarrow ()$

Thus there are two ways to derive (). Thus the context-free grammar is ambiguous. [5 points]

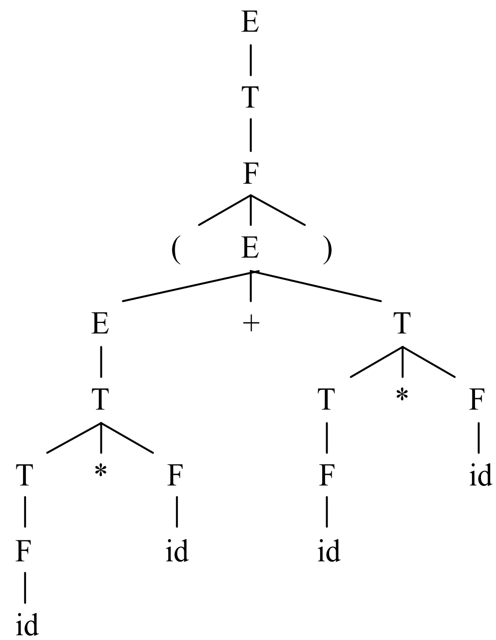
Example of unambiguous grammar: $G=(\{(,),S,T\},\{(,)\},R,S)$ where $R=\{S \rightarrow T, S \rightarrow e, T \rightarrow (), T \rightarrow (T), T \rightarrow TT\}$. [5 points]

Problem 3.2.4(b) [5+5=10 points]

id+(id+id)*id



(id*id+id*id)



Problem 3.3.1 [3×1+3×1+2×1+3×1+4=15 points]

(a) There are 3 possible computations on input *aba*. [3×1=3 points]

$(s,aba,e) \vdash^*_M (f,ba,e)$

$(s,aba,e) \vdash^*_M (s,ba,a) \vdash^*_M (s,a,aa) \vdash^*_M (s,e,aaa)$

$(s,aba,e) \vdash^*_M (s,ba,a) \vdash^*_M (s,a,aa) \vdash^*_M (f,e,aa)$

(b) There are 3 possible computations on input *aa*. [3×1=3 points]

$(s,aa,e) \vdash^*_M (f,a,e)$

$(s,aa,e) \vdash^*_M (s,a,a) \vdash^*_M (f,e,a)$

$(s,aa,e) \vdash^*_M (s,a,a) \vdash^*_M (s,e,aa)$

There are 2 possible computations on input *abb*. [2×1=2 points]

$(s,abb,e) \vdash^*_M (f,bb,e)$

$(s,abb,e) \vdash^*_M (s,bb,a) \vdash^*_M (s,b,aa) \vdash^*_M (s,e,aaa)$

All *aba*, *aa* and *abb* are not accepted.

All *baa*, *bab* and *baaaa* are accepted. [3×1=3 points]

$(s,baa,e) \vdash^*_M (s,aa,a) \vdash^*_M (f,a,a) \vdash^*_M (f,e,e)$

$(s,bab,e) \vdash^*_M (s,ab,a) \vdash^*_M (f,b,a) \vdash^*_M (f,e,e)$

$(s,baaaa,e) \vdash^*_M (s,aaaa,a) \vdash^*_M (s,aaa,aa) \vdash^*_M (f,aa,aa) \vdash^*_M (f,a,a) \vdash^*_M (f,e,e)$

(c) $L=\{xay: x,y \in \{a,b\}^*, |x|=|y|\}$ [4 points]

Problem 3.3.2 (b) [10 points]

Example of pushdown automata accepts $\{a^m b^n: m \le 2n\}$:

$M=\{K,\Sigma,T,\triangle,q,F\}$, where $K=\{q,r\}$, $\Sigma=\{a,b\}$, $T=\{a\}$, $F=\{r\}$, and

$\triangle=\{ ((q,a,e),(q,aa)), ((q,a,e),(q,a)), ((q,e,e),(r,e)), ((r,b,a),(r,e)) \}$