CMSC452 Elementary Theory of Computation, Fall 2001

Sketch of Solutions: Homework 5

Note: 5 partial credits are given to each graded unit for minor errors.

Problem 3.4.1 [10+10=20 points]

The corresponding machine is $M=\{\{p,q\},\{(,)\},\{(,),S\},\triangle,p,\{q\}\}$, where $\triangle =\{((p,e,e),(q,S)),((q,e,S),(q,SS)),((q,e,S),(q,(S))),((q,e,S),(q,e)),((q,(,(),(q,e)),((q,),)),(q,e))\}$ [10 ponits]

Then acceptance of the input string (()()): $(p,(()()),e) \vdash_M (q,(()()),S) \vdash_M (q,(()()),(S)) \vdash_M (q,()()),S)) \vdash_M (q,()()),SS)) \vdash_M (q,()()),(S)S)) \vdash_M (q,)()),S)S)) \vdash_M (q,)()),)S)) \vdash_M (q,()),S)) \vdash_M (q,()),(S))) \vdash_M (q,)),S))) \vdash_M (q,)),))) \vdash_M (q,),)) \vdash_M (q,e,e)$

Problem 3.4.4 [10+10=20 points]

Each transition relation can be replaced to be some others satisfying the desired constraints with identical effect by the following two methods repeatedly. Thus for each pushdown automaton M, there is another equivalent automaton M' satisfying the desired constraints.

1. Given transition relation $((p,\alpha,\beta),(q,\gamma))$, if both $\beta\neq e$ and $\gamma\neq e$, replace it by $((p,\alpha,\beta),(r,e))$ and $((r,e,e),(q,\gamma))$, where r is a new state. [10 points]
2. For each transition relation $((p,\alpha,\beta),(q,\gamma))$, if $|\gamma|>1$, it can be replaced by $((p,\alpha,\beta),(r, \gamma_1))$ and $((r,e,e),(q,\gamma_2))$, where $\gamma=\gamma_1\gamma_2$ with $|\gamma_1|\geq 1$ and $|\gamma_2|\geq 1$ and r is a new state. After $|\beta|$-1 such operations, the transition relation can be replace by $|\beta|$ others with unit β's. Similarly, Each transition relation with $|\gamma|>1$ can be replaced by some others with unit γ's. [10 points]

Problem 3.5.1 (a) [10 points]

Let $L_1=\{a\}^+\{a^nb^n: n\in N\}=\{a^mb^n: m>n\}$. Then L_1 is context-free since it is the concatenation of two context-free languages. Similarly, $L_2=\{a^nb^n: n\in N\}\{b\}^+=\{a^mb^n: m>n\}$ is also context-free. Finally, $\{a^mb^n: m\neq n\}=\{a^mb^n: m>n\}\cup\{a^mb^n: m>n\}= L_1\cup L_2$ is also context-free.

Problem 3.5.2 (c) [10 points]

Suppose $L=\{www: w\in\Sigma^*\}$ is context-free. By theorem 3.5.3, there is $k>0$ such that for any $w\in L$ with $|w|\geq k$, w can be represented by $uvxyz$ where $u,v,x,y,z\in\Sigma^*$, $|vxy|\leq k$ $vy\neq e$ such that $uv^nxy^nz\in L$ for $n\geq 0$.

1. Consider *the* string $w=a^kba^kba^kb$ with $|w|\geq k$. It can be represented by $w=uvxyz$ as above.
2. Neither v nor y can contain b. Otherwise, $uv^nxy^nz\notin L$ for $n\geq 4$.
3. Thus $v,y\in\{a^*: |a^*|\leq k\}$. Let $v=a^p$ and $y=a^q$. for $p,q\leq k$. Suppose v is in the first a's and y is the second a's. Then $uv^2xy^2z=a^{k+p}ba^{k+q}ba^kb\notin L$. In general, other cases will result in $uv^2xy^2z=a^{k+r}ba^{k+s}ba^{k+t}b$ such that $r,s,t\in\{0,p,q,p+q\}$ and $r+s+t=p+q$. All the possibilities cannot be in L. Thus L is not context-free by contradiction.

Problem 3.5.14 (b)(d) [10+10=20 points]

(b) $\{a^m b^n c^p$: $m \neq n$ or $n \neq p$ or $p \neq m\}$ is context-free. $\{a^m b^n c^p$: $m \neq n\}$ is context-free because it is essentially of the form of problem 3.5.1. Similarly, $\{a^m b^n c^p$: $n \neq p\}$ and $\{a^m b^n c^p$: $p \neq m\}$ are context-free. Thus, $\{a^m b^n c^p$: $m \neq n$ or $n \neq p$ or $p \neq m\} = \{a^m b^n c^p$: $m \neq n\} \cup \{a^m b^n c^p$: $n \neq p\} \cup \{a^m b^n c^p$: $p \neq m\}$, a union of context-free languages, is also context-free. [10 points]

(d) L=$\{w \in \{a,b,c\}^*$: w does not contain equal numbers of occurrences of a, b, and $c\}$ is context-free. Let $L_1=\{w \in \{a,b,c\}^*$: the number of occurrence of a is different from that of $b\}$, $L_2=\{w \in \{a,b,c\}^*$: the number of occurrence of b is different from that of $c\}$, and $L_3=\{w \in \{a,b,c\}^*$: the number of occurrence of c is different from that of $a\}$. Since $L= L_1 \cup L_2 \cup L_3$, a union of context-free languages, L is context-free.

Problem 3.5.15 [10+10=20 points]

$L-R=L \cap R^c$ is context-free since it is the intersection of context-free language L and regular language $R^c=\Sigma^*-R$. [10 points]

R-L is not necessarily context-free. For example, let $R=\Sigma^*$ which is regular. Then $R-L=L^c$ needs not to be context-free since context-free languages are not closed under complementation. [10 points]