CMSC452 Elementary Theory of Computation, Fall 2001

Sketch of Solutions: Homework 6

Note: 5 partial credits are given to each graded unit for minor errors.

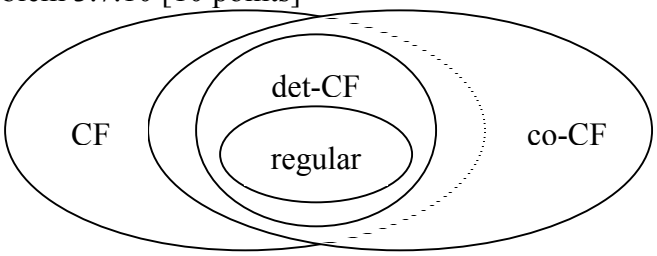
Problem 3.7.1 (a) [10 points]

The following deterministic pushdown automaton accepts the context-free language L\$, where $L=\{a^m b^n: m\neq n\}$. Thus, L is deterministic context-free.

M={K,∑,Γ,△,*s*,F}, where
K={*s*,*p*,*q*,*r*,*t*,*f*}
∑={*a*,*b*,\$}
Γ={*a*,*x*}
F={*f*}

△={((*s*,*e*,*e*),(*p*,*x*)),((*p*,*a*,*e*),(*p*,*a*)),((*p*,\$,*a*),(*r*,*e*)),((*p*,*b*,*x*),(*t*,*e*)),((*p*,*b*,*a*),(*q*,*e*)),
((*q*,*b*,*a*),(*q*,*e*)),((*q*,\$,*a*),(*r*,*e*)),((*q*,*b*,*x*),(*t*,*e*)),((*t*,*b*,*e*),(*t*,*e*)),((*t*,\$,*e*),(*f*,*e*)),((*r*,*e*,*a*),(*r*,*e*)),
((*r*,*e*,*x*),(*f*,*e*))}
Note: ((*p*,\$,*a*),(*r*,*e*)) is for $n=0$, ((*p*,*b*,*x*),(*t*,*e*)) is for $m=0$,
((*q*,\$,*a*),(*r*,*e*)) is for $m>n\neq 0$, and ((*q*,*b*,*x*),(*t*,*e*)) is for $n>m\neq 0$.

Problem 3.7.10 [10 points]



CF: context-free
co-CF:: complements of context-free
det-CF: deterministic context-free

Problem 4.1.1 [10+10=20 points]

(a) $(q_0,\blacktriangleright aabbba) \vdash_M (q_1,\blacktriangleright \underline{b}abbba) \vdash_M (q_0,\blacktriangleright b\underline{a}bbba) \vdash_M (q_1,\blacktriangleright b\underline{b}bbba)$
$\vdash_M (q_0,\blacktriangleright bb\underline{b}bba) \vdash_M (q_1,\blacktriangleright bb\underline{a}bba) \vdash_M (q_0,\blacktriangleright bba\underline{b}ba) \vdash_M (q_1,\blacktriangleright bba\underline{a}ba)$
$\vdash_M (q_0,\blacktriangleright bbaa\underline{b}a) \vdash_M (q_1,\blacktriangleright bbaa\underline{a}a) \vdash_M (q_0,\blacktriangleright bbaaa\underline{a}) \vdash_M (q_1,\blacktriangleright bbaaa\underline{b})$
$\vdash_M (q_0,\blacktriangleright bbaaab\sqcup) \vdash_M (h,\blacktriangleright bbaaab\sqcup)$ [10 points]

(b) This Turing machine moves right, and swaps all *a*s and *b*s as it goes, until reaching a blank. When reaching a blank, it halts. [10 points]

Problem 4.1.4 [10+10=20 points]

M scans to the left, replacing every other *a* with a ⊔. The first *a* being replaced is the second to last *a*, the one to the left of the initially being scanned *a*. [10 points]
If n is even, then M halts on the ⊔ adjacent to the ►. If *n* is odd, then M finishes by looping forever on that ⊔. [10 points]

Problem 4.1.6 (a) [10 points]

According to definition 4.1.2, Turing machine configurations are defined to be triples, whereas all of these are quadruples. Thus none of these can be configurations.

If Turing machine configurations are expressed as quadruples in Figure 4.2, only (i), (iii), and (viii) could be configurations. Note that the third element represents the place which the head points to, so it cannot be an *e*.

Problem 4.1.7 [10 points]

M={K,Σ,δ,*s*,H} where K={q_0,q_1,*h*}, Σ={►,␣,*a*,*b*}, H={*h*} and δ is defined as follows:

q	σ	$\delta(q,\sigma)$
q_0	*a*	$(q_1,\rightarrow)$
q_0	*b*	$(q_0,\rightarrow)$
q_0	␣	$(q_0,\rightarrow)$
q_0	►	$(q_0,\rightarrow)$
q_1	*a*	$(h,\rightarrow)$
q_1	*b*	$(q_0,\rightarrow)$
q_1	␣	$(q_0,\rightarrow)$
q_1	►	$(q_0,\rightarrow)$

Problem 4.1.10 [10+10=20 points]

This Turing machine moves the head to the right, remembering the first and second nonblank symbols, denoted by *a* and *b*, respectively. [10 points]

After that, it continues to the right, replacing the first and second blanks it encounters by *a* and *b*, respectively. [10 points]