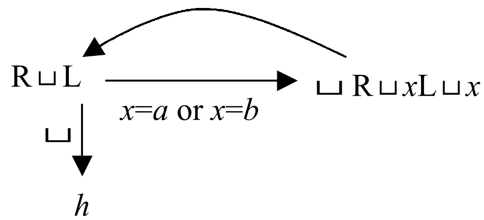
CMSC452 Elementary Theory of Computation, Fall 2001

Sketch of Solutions: Homework 7

Note: 5 or 10 partial credits are given to each graded unit for minor errors.

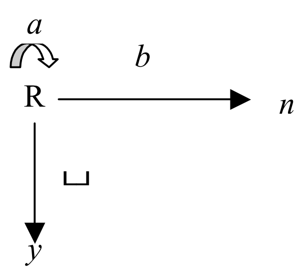
Problem 4.2.1 [15 points]

$f(w)=ww^R$ from strings in $\{a,b\}^*$



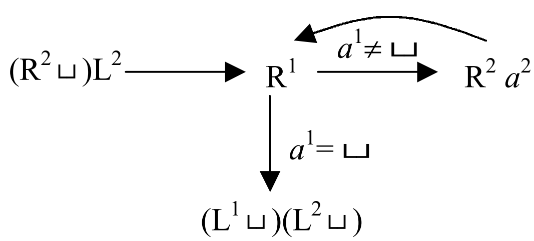
Problem 4.2.2 (d) [15 points]

L(M)=$\{a\}^*$ over $\{a,b\}$



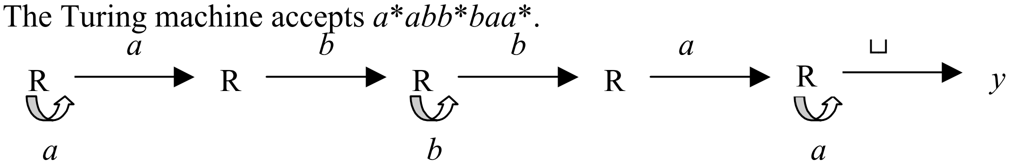
Problem 4.3.3 [15 points]

Let R^1, L^1 and R^2, L^2 denote the movement of the first and second heads, respectively, a^1 and a^2 denote the reading/writing of the first and second head, also respectively. The following 2-head Turing machine computes $f(w)=ww$. Note that $R^2\sqcup$ means that head 2 reads rightward until blank.



Problem 4.5.1 (a) [15 points]

The Turing machine accepts $a^*abb^*baa^*$.



Problem 4.5.3 (for union) [15 points]

Given Turing machines $M_1=\{K_1, \Sigma_1, \delta_1, s_1, \{y,n\}\}$ and $M_2=\{K_2, \Sigma_2, \delta_2, s_2, \{y,n\}\}$, define non-deterministic Turing machine $M=\{K, \Sigma, \delta, s, \{y,n\}\}$ where $K= K_1\cup K_2\cup\{s\}$, $\Sigma=\Sigma_1\cup\Sigma_2$, $\delta=\delta_1\cup\delta_2\cup\{\delta(s,\sqcup)\to(s_1,\sqcup), \delta(s,\sqcup)\to(s_2,\sqcup)\}$, and $s\notin K_1\cup K_2$ is a new symbol. Then $L(M)=L(M_1)\cup L(M_2)$.

Problem 4.6.1 (a) [15 points]

$S\Rightarrow ABCS\Rightarrow ABCABCS\Rightarrow ABCABCABCS\Rightarrow ABCABCABCT_c\Rightarrow$
$ABCABACBCT_c\Rightarrow ABCABABCCT_c\Rightarrow$
$ABACBABCCT_c\Rightarrow ABABCABCCT_c\Rightarrow ABABACBCCT_c\Rightarrow ABABABCCCT_c\Rightarrow$
$ABAABBCCCT_c\Rightarrow AABABBCCCT_c\Rightarrow AAABBBCCCT_c\Rightarrow$
$AAABBBCCT_cc\Rightarrow AAABBBCT_ccc\Rightarrow AAABBBT_bccc\Rightarrow$
$AAABBT_bbccc\Rightarrow AAABT_bbbccc\Rightarrow AAAT_abbbccc$
$AAAT_abbbccc\Rightarrow AAT_aabbbccc\Rightarrow AT_aaabbbccc\Rightarrow T_aaaabbbccc\Rightarrow aaabbbccc$

Problem 4.6.2 (c) [10 points]

The grammar $G=\{V, \Sigma, R, S\}$ accepts a^{n^2}, where
$V=\{a,S,T,B,C,\$\}$
$\Sigma=\{a\}$
$R=\{S\to\$T\$, T\to BTC, T\to e, BC\to CaB, aC\to Ca, Ba\to aB, \$C\to\$, B\$\to\$, \$\to e\}$
Idea: How many swaps does bubble sort take to sort B^nC^n to be C^nB^n? The answer is n^2. In this rule set, an a is generated for swapping BC, and swapping Ba or aC is free (i.e. No a is generated). Thus, there are n^2 as are generated with B^nC^n.