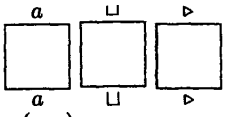
CMSC452 Elementary Theory of Computation, Fall 2001

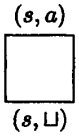
Sketch of Solutions: Homework 9

Problem 5.6.1

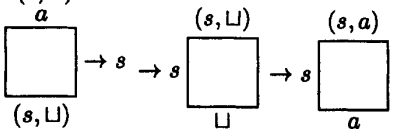
The tiles communicate unchanged symbols:



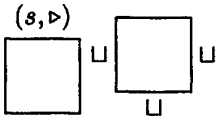
The tile indicates stationary changes of state:



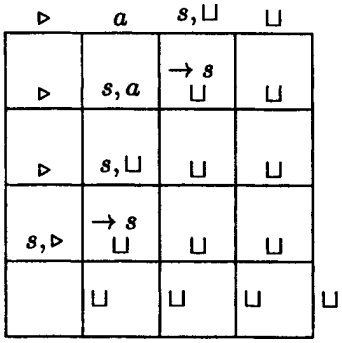
The tiles indicate head motion to the right:



There are no tiles for head movement to the left for this TM; the tiles are required always:



The diagram indicates the first four boxes of the first four rows of a tiling of the plane using above tiles:



Problem 5.7.1

1. (⇒) For each recursively enumerable L, there is a deterministic Turing machine which enumerates L. Since each deterministic Turing machine is also a non-deterministic Turing machine, L is enumerated by a non-deterministic Turing machine,
2. (⇐) Suppose that L is enumerated by some non-deterministic Turing machine M. By theorem 4.5.1, there is a deterministic Turing machine with the same function. That is, we can run a breadth-first simulation of all possible computations of M. For any given string w accepted by M, the simulator will also accept w.

Problem 5.7.4 (a)

Given a string $w \in \Sigma^*$, define set $V=\{v: v \in \Sigma^*, |v| \le |w|\}$, and directed graph $G=\{V,E\}$ where $E=\{(x,y): x \Rightarrow_G y\}$. Because G is finite, the reachability of any two vertices in V can be computed in finite time. So $\Rightarrow_G^*$ can be completely determined in finite time. Because context-sensitive grammar has the property that $x \Rightarrow_G y$ implies $|x| \le |y|$, it suffices to consider only V, the strings of length $|w|$ or less. Thus, each $w \in \Sigma^*$ can be accepted or rejected in finite time. Every context-sensitive language is recursive.

Problem 5.7.7 (a)

For each case of (a)-(e), it can be shown that the class is a subset of all recursively enumerable languages and there exists one recursively enumerable language in the class and another one not in the class. By Rice's theorem, each of them is undecidable.

For (a), let C be the class of finite languages.

1. Because every finite language is recursive, C is a subset of the class of all recursively enumerable languages.
2. The language $L=\{ww^R: w\in\{a,b\}^*\}$ is recursively enumerable but not regular. Note that all the finite languages are regular Thus, L is not finite and $L\notin C$. C is a proper subset of recursively enumerable languages.
3. Since $\sum^*$ is regular for any given $\sum$, C is nonempty

By Rice's theorem, to tell whether a given Turing machine M semidecides a language in the class of finite languages is undecidable.