

Program the following 13 functions in LISP. Make sure you test them thoroughly. Sample data will be mailed to you. Turn in a listing of your program and the results of applying the test data.

1. Given two sets of atoms x and y represented as lists, write functions `union[x, y]`, `intersection[x, y]` and `set_difference[x, y]`, for their union $x \cup y$, intersection $x \cap y$, and set difference $x \setminus y$, respectively. Use the function `member[n, x]` defined below, which may also be written as $n \in x$:

```
member(x, u) = if null u then nil
               else if car u eq x then t
               else member(x, cdr u)
```

For example, $(A\ B\ C) \cup (B\ C\ D) = (A\ B\ C\ D)$, $(A\ B\ C) \cap (B\ C\ D) = (B\ C)$, and $(A\ B\ C) \setminus (B\ C\ D) = (A)$.

Pay attention to getting correct the trivial (i.e., base) cases in which some of the arguments are `nil`. In general, it is important to understand clearly the trivial cases of functions.

2. Given an integer n and a list l of integers sorted in increasing order, write a function `merge[n, l]` which inserts n in its proper place in l . For example, `merge[3, '(2 4)] = (2 3 4)`, and `merge[3, '(2 3)] = (2 3 3)`.
3. Given two sets of atoms x and y represented as ordered lists containing no duplicates, write functions `ounion[x, y]`, `ointersection[x, y]` and `oset_difference[x, y]` giving the union, intersection, and set difference, respectively, of x and y ; the result is wanted as an ordered list.

Note that computing these functions of unordered lists takes a number of comparisons proportional to the square of the number of elements of a typical list, while for ordered lists, the number of comparisons is proportional to the number of elements.

4. Using `merge`, write a function named `sort[l]` that transforms an unordered list l into an ordered list. Your algorithm should repeatedly invoke the `merge` function starting with an empty list, thereby running in $O(n^2)$ time for a list of n elements.
5. Write a predicate `occur[a, s]` to indicate whether an atom a occurs in a given s-expression s , e.g., `occur[B, '((A B) . C)] = t`.
6. Write a function `num_occur[a, s]` that indicates how many times an atom a occurs in an s-expression s , e.g., `num_occur[B, '((A B) . C)] = 1`.
7. Write a function `nodups[s]` to make a list without duplications of the atoms occurring in an s-expression s , e.g., `nodups['((A . B) . (C . A))]` = `(A B C)`.

8. Write a function `multiplicity[s]` that indicates which atoms occur more than once in an s-expression `s`. The result should be in the form of a list of pairs (i.e., an assoc-list), where each pair consists of the atom that occurs more than once and its multiplicity, e.g., `multiplicity['((A . B) . (C . A))]` = `((A . 2))`.
9. Write a predicate `multi_occur_sexpr[x, y]` that indicates whether or not an s-expression `x` has more than one occurrence of an s-expression `y` as a sub-expression, e.g., `multi_occur_sexpr['((A . B) . (C . (A . B)))`, `(A . B)]` = `t`.