Fall 2003 **Programming Project #3** CMSC 420

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Program the following 13 functions in LISP. Make sure you test them thoroughly. Sample data will be mailed to you. Turn in a listing of your program and the results of applying the test data.

1. Given two sets of atoms `x` and `y` represented as lists, write functions `union[x, y]`, `intersection[x, y]` and `set_difference[x, y]`, for their union $\mathtt{x} \cup \mathtt{y}$, intersection $\mathtt{x} \cap \mathtt{y}$, and set difference $\mathtt{u} \setminus \mathtt{y}$, respectively. Use the function `member[n, x]` defined below, which may also be written as $\mathtt{n} \in \mathtt{x}$:

   ```
   member(x, u) = if null u then nil
                  else if car u eq x then t
                  else member(x,cdr u)
   ```

   For example, `(A B C)` $\cup$ `(B C D)` = `(A B C D)`, `(A B C)` $\cap$ `(B C D)` = `(B C)`, and `(A B C)` $\setminus$ `(B C D)` = `(A)`.

   Pay attention to getting correct the trivial (i.e., base) cases in which some of the arguments are `nil`. In general, it is important to understand clearly the trivial cases of functions.

2. Given an integer `n` and a list `l` of integers sorted in increasing order, write a function `merge[n, l]` which inserts `n` in its proper place in `l`. For example, `merge[3, '(2 4)]` = `(2 3 4)`, and `merge[3, '(2 3)]` = `(2 3 3)`.

3. Given two sets of atoms `x` and `y` represented as ordered lists containing no duplicates, write functions `ounion[x, y]`, `ointersection[x, y]` and `oset_difference[x, y]` giving the union, intersection, and set difference, respectively, of `x` and `y`; the result is wanted as an ordered list.

   Note that computing these functions of unordered lists takes a number of comparisons proportional to the square of the number of elements of a typical list, while for ordered lists, the number of comparisons is proportional to the number of elements.

4. Using `merge`, write a function named `sort[l]` that transforms an unordered list `l` into an ordered list. Your algorithm should repeatedly invoke the `merge` function starting with an empty list, thereby running in $O(n^2)$ time for a list of $n$ elements.

5. Write a predicate `occur[a, s]` to indicate whether an atom `a` occurs in a given s-expression `s`, e.g., `occur[B, '((A B) . C)]` = `t`.

6. Write a function `num_occur[a, s]` that indicates how many times an atom `a` occurs in an s-expression `s`, e.g., `num_occur[B, '((A B) . C)]` = `1`.

7. Write a function `nodups[s]` to make a list without duplications of the atoms occurring in an s-expression `s`, e.g., `nodups['((A . B) . (C . A))] = (A B C)`.

8. Write a function `multiplicity[s]` that indicates which atoms occur more than once in an s-expression `s`. The result should be in the form of a list of pairs (i.e., an assoc-list), where each pair consists of the atom that occurs more than once and its multiplicity, e.g., `multiplicity['((A . B) . (C . A))] = ((A . 2))`.

9. Write a predicate `multi_occur_sexpr[x, y]` that indicates whether or not an s-expression `x` has more than one occurrence of an s-expression `y` as a sub-expression, e.g., `multi_occur_sexpr['((A . B) . (C . (A . B))), (A . B)] = t`.