Learning

R&N: ch 19, ch 20

based on material from Ray Mooney, Daphne Koller, Kevin Murphy, Marie desJardins

Types of Learning

- Supervised Learning - classification, prediction
- Unsupervised Learning – clustering, segmentation, pattern discovery
- Reinforcement Learning – learning MDPs, online learning

Supervised Learning

- Someone gives you a bunch of examples, telling you what each one is
- Eventually, you figure out the mapping from properties (**features**) of the examples to their type (**classification**)

Supervised Learning

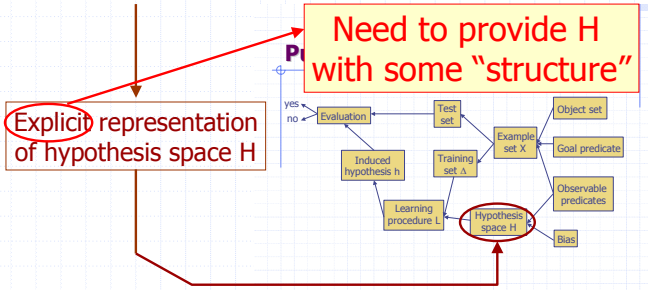
- Logic-based approaches:
  - learn a function f(X) $\rightarrow$ (true,false)
- Statistical/Probabilistic approaches
  - Learn a probability distribution p(Y|X)

Outline

- Logic-based Approaches
  - Decision Trees (last time)
  - Version Spaces (today)
  - PAC learning (we won't cover, ☹)
- Instance-based approaches
- Statistical Approaches

Predicate-Learning Methods

- Version space



Version Spaces

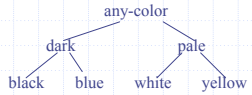
- The "version space" is the set of all hypotheses that are consistent with the training instances processed so far.
- An algorithm:
  - V := H ;; the version space V is ALL hypotheses H
  - For each example e:
    - Eliminate any member of V that disagrees with e
    - If V is empty, FAIL
  - Return V as the set of consistent hypotheses

Version Spaces: The Problem

- PROBLEM: V is huge!!
- Suppose you have N attributes, each with k possible values
- Suppose you allow a hypothesis to be any disjunction of instances
- There are k^N possible instances $\rightarrow |H| = 2^{k^N}$
- If N=5 and k=2, $|H| = 2^{32}$!!

Version Spaces: The Tricks

- First Trick: Don't allow arbitrary disjunctions
  - Organize the feature values into a hierarchy of allowed disjunctions, e.g.



  - Now there are only 7 "abstract values" instead of 16 disjunctive combinations (e.g., "black or white" isn't allowed)
- Second Trick: Define a partial ordering on H ("general to specific") and only keep track of the upper bound and lower bound of the version space
- RESULT: An incremental, possibly efficient algorithm!

Rewarded Card Example

(r=1) v … v (r=10) v (r=J) v (r=Q) v (r=K) ⇔ ANY-RANK(r)
(r=1) v … v (r=10) ⇔ NUM(r)
(r=J) v (r=Q) v (r=K) ⇔ FACE(r)
(s=♠) v (s=♣) v (s=♦) v (s=♥) ⇔ ANY-SUIT(s)
(s=♠) v (s=♣) ⇔ BLACK(s)
(s=♦) v (s=♥) ⇔ RED(s)

A hypothesis is any sentence of the form:

R(r) ∧ S(s) ⇔ IN-CLASS([r,s])

where:

- R(r) is ANY-RANK(r), NUM(r), FACE(r), or (r=j)
- S(s) is ANY-SUIT(s), BLACK(s), RED(s), or (s=k)

Simplified Representation

For simplicity, we represent a concept by rs, with:

- r ∈ {a, n, f, 1, …, 10, j, q, k}
- s ∈ {a, b, r, ♣, ♠, ♦, ♥}

For example:

- n♠ represents:
  NUM(r) ∧ (s=♠) ⇔ IN-CLASS([r,s])
- aa represents:
  ANY-RANK(r) ∧ ANY-SUIT(s) ⇔ IN-CLASS([r,s])

Extension of a Hypothesis

The extension of a hypothesis h is the set of objects that satisfies h

Examples:

- The extension of f♠ is: {j♠, q♠, k♠}
- The extension of aa is the set of all cards

More General/Specific Relation

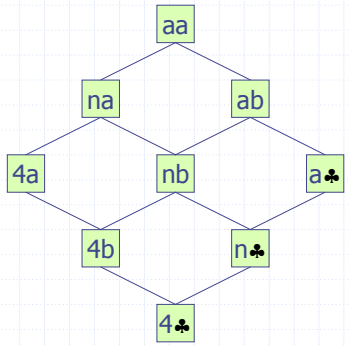
- Let h_1 and h_2 be two hypotheses in H
- h_1 is more general than h_2 iff the extension of h_1 is a proper superset of the extension of h_2

Examples:
- aa is more general than f♦
- f♥ is more general than q♥
- fr and nr are not comparable

More General/Specific Relation

- Let h_1 and h_2 be two hypotheses in H
- h_1 is more general than h_2 iff the extension of h_1 is a proper superset of the extension of h_2
- The inverse of the "more general" relation is the "more specific" relation
- The "more general" relation defines a partial ordering on the hypotheses in H

Example: Subset of Partial Order



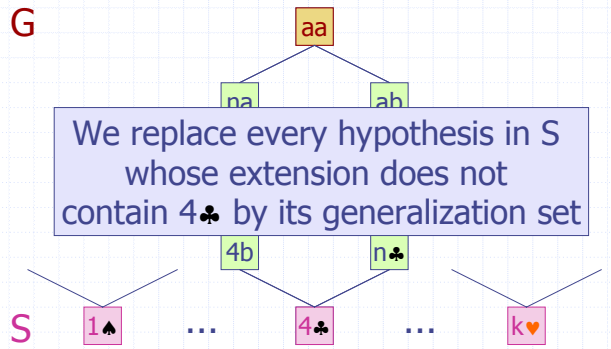
G-Boundary / S-Boundary of V

- A hypothesis in V is most general iff no hypothesis in V is more general
- G-boundary G of V: Set of most general hypotheses in V

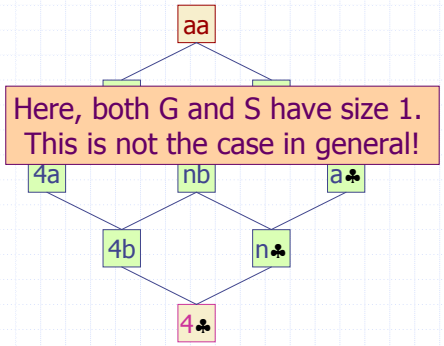
G-Boundary / S-Boundary of V

- A hypothesis in V is most general iff no hypothesis in V is more general
- G-boundary G of V: Set of most general hypotheses in V
- A hypothesis in V is most specific iff no hypothesis in V is more specific
- S-boundary S of V: Set of most specific hypotheses in V

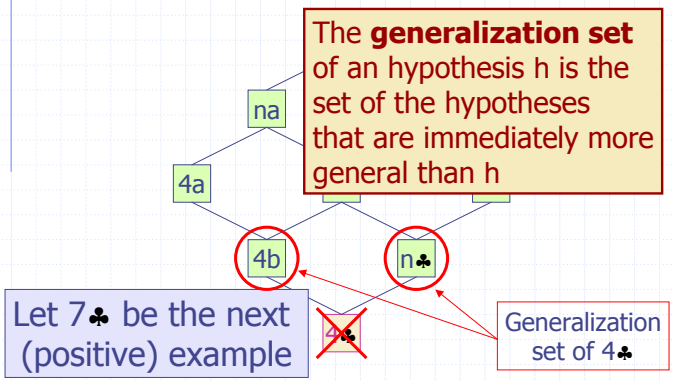
Example: G-/S-Boundaries of V



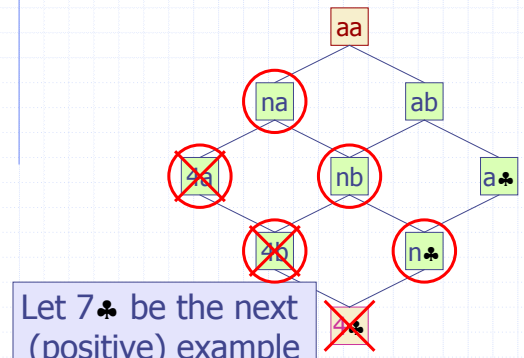
Example: G-/S-Boundaries of V



Example: G-/S-Boundaries of V



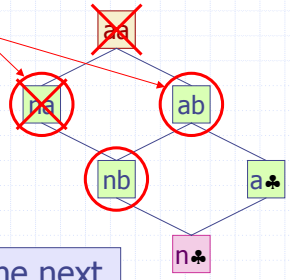
Example: G-/S-Boundaries of V



Let 7♣ be the next (positive) example

Example: G-/S-Boundaries of V

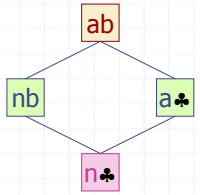
Specialization set of aa



Let 5♥ be the next (negative) example

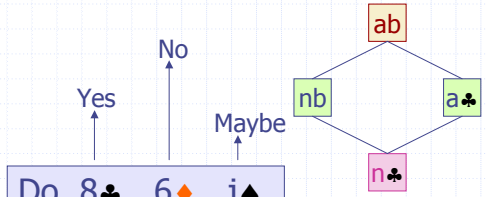
Example: G-/S-Boundaries of V

G and S, and all hypotheses in between form exactly the version space



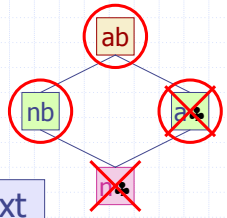
Example: G-/S-Boundaries of V

At this stage …



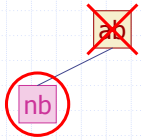
Do 8♣, 6♦, j♠ satisfy CONCEPT?

Example: G-/S-Boundaries of V



Let 2♠ be the next (positive) example

Example: G-/S-Boundaries of V



Let j♠ be the next (negative) example

Example: G-/S-Boundaries of V

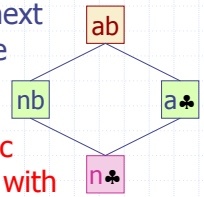
+ 4♣ 7♣ 2♠
– 5♥ j♠

nb

NUM(r) ∧ BLACK(s) ⇔ IN-CLASS([r,s])

Example: G-/S-Boundaries of V

Let us return to the version space …
… and let 8♣ be the next (negative) example

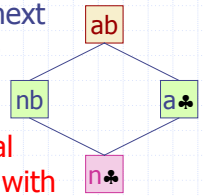


The only most specific hypothesis disagrees with this example, so no hypothesis in H agrees with all examples

Example: G-/S-Boundaries of V

Let us return to the version space …

… and let j♥ be the next (positive) example



The only most general hypothesis disagrees with this example, so no hypothesis in H agrees with all examples

Version Space Update

1. x ← new example
2. If x is positive then
   (G,S) ← POSITIVE-UPDATE(G,S,x)
3. Else
   (G,S) ← NEGATIVE-UPDATE(G,S,x)
4. If G or S is empty then return failure

POSITIVE-UPDATE(G,S,x)

1. Eliminate all hypotheses in G that do not agree with x

POSITIVE-UPDATE(G,S,x)

1. Eliminate all hypotheses in G that do not agree with x
2. Minimally generalize all hypotheses in S until they are consistent with x

Using the generalization sets of the hypotheses

POSITIVE-UPDATE(G,S,x)

1. Eliminate all hypotheses in G that do not agree with x
2. Minimally generalize all hypotheses in S until they are consistent with x
3. Remove from S every hypothesis that is neither more specific than nor equal to a hypothesis in G

This step was not needed in the card example

POSITIVE-UPDATE(G,S,x)

1. Eliminate all hypotheses in G that do not agree with x
2. Minimally generalize all hypotheses in S until they are consistent with x
3. Remove from S every hypothesis that is neither more specific than nor equal to a hypothesis in G
4. Remove from S every hypothesis that is more general than another hypothesis in S
5. Return (G,S)

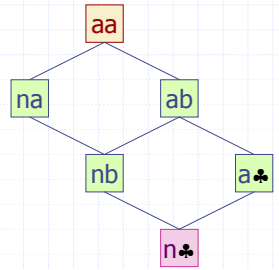
NEGATIVE-UPDATE(G,S,x)

1. Eliminate all hypotheses in S that do not agree with x
2. Minimally specialize all hypotheses in G until they are consistent with x
3. Remove from G every hypothesis that is neither more general than nor equal to a hypothesis in S
4. Remove from G every hypothesis that is more specific than another hypothesis in G
5. Return (G,S)

Example-Selection Strategy (aka Active Learning)

- Suppose that at each step the learning procedure has the possibility to select the object (card) of the next example
- Let it pick the object such that, whether the example is positive or not, it will eliminate one-half of the remaining hypotheses
- Then a single hypothesis will be isolated in O(log |H|) steps

Example



- 9♣?
- j♥?
- j♣?

Example-Selection Strategy

- Suppose that at each step the learning procedure has the possibility to select the object (card) of the next example
- Let it pick the object such that, whether the example is positive or not, it will eliminate one-half of the remaining hypotheses
- Then a single hypothesis will be isolated in O(log |H|) steps
- But picking the object that eliminates half the version space may be expensive

Noise

- If some examples are misclassified, the version space may collapse
- Possible solution:
  Maintain several G- and S-boundaries, e.g., consistent with all examples, all examples but one, etc…

Version Spaces

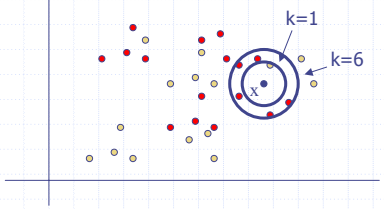
- Useful for understanding logical (consistency-based) approaches to learning
- Practical if strong hypothesis bias (concept hierarchies, conjunctive hypothesis)
- Don't handle noise well

Statistical Approaches

- Instance-based Learning (20.4)
- Statistical Learning (20.1)
- Neural Networks (20.5) on Dec. 2

Nearest Neighbor Methods

- To classify a new input vector x, examine the k-closest training data points to x and assign the object to the most frequently occurring class



Issues

- Distance measure
  - Most common: euclidean
  - Better distance measures: normalize each variable by standard deviation
- Choosing k
  - Increasing k reduces variance, increases bias
- For high-dimensional space, problem that the nearest neighbor may not be very close at all!
- Memory-based technique. Must make a pass through the data for each classification. This can be prohibitive for large data sets.
- Indexing the data can help; for example KD trees

Bayesian Learning

Example: Candy Bags

- Candy comes in two flavors: cherry (☺) and lime (☹)
- Candy is wrapped, can't tell which flavor until opened
- There are 5 kinds of bags of candy:
  - H_1= all cherry
  - H_2= 75% cherry, 25% lime
  - H_3= 50% cherry, 50% lime
  - H_4= 25% cherry, 75% lime
  - H_5= 100% lime
- Given a new bag of candy, predict H
- Observations: D_1, D_2 , D_3, …

Bayesian Learning

- Calculate the probability of each hypothesis, given the data, and make prediction weighted by this probability (i.e. use all the hypothesis, not just the single best)

$$P(h_i \mid d) = \frac{P(d \mid h_i)P(h_i)}{P(d)} = \alpha P(d \mid h_i)P(h_i)$$

- Now, if we want to predict some unknown quantity X

$$P(X \mid d) = \sum_i P(X \mid h_i)P(h_i \mid d)$$

Bayesian Learning cont.

- Calculating P(h|d)

$$P(h_i \mid d) \propto P(d \mid h_i)P(h_i)$$

likelihood prior

- Assume the observations are i.i.d.—independent and identically distributed

$$P(d \mid h_i) = \prod_j P(d_j \mid h_i)$$

Example:

- Hypothesis Prior over h_1, …, h_5 is {0.1,0.2,0.4,0.2,0.1}
- Data:

- Q1: After seeing d_1, what is $P(h_i|d_1)$?
- Q2: After seeing d_1, what is $P(d_2=$ ●$|d_1)$?