CMSC 250 Fall 2004 — Homework 2 Answer

Due Wed., Sept. 15 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. For each of the following statements, give its converse, inverse, and contrapositive in English sentences; be sure to label the three parts of each answer. You may change verb tenses to make your answers sound better.

   (a) If people turn to look at you on the street, you are not well dressed.
   Answer:
   - Converse: If you are not well dressed, then people turn to look at you on the street.
   - Inverse: If people do not turn to look at you on the street, then you are well dressed.
   - Contrapositive: If you are well dressed, then people do not turn to look at you on the street.

   (b) If you want anything done well, do it yourself.
   Answer:
   - Converse: If you do anything yourself, then you want it done well.
   - Inverse: If you do not want anything done well, then you do not do it yourself.
   - Contrapositive: If you do not do anything yourself, you do not want it done well.

   (c) If you're not part of the solution, you're part of the precipitate.
   Answer:
   - Converse: If you're part of the precipitate, you're not part of the solution.
   - Inverse: If you're part of the solution, you're not part of the precipitate.
   - Contrapositive: If you're not part of the precipitate, you're part of the solution.

   (d) The gift of grace can be yours only if you'll reach out and take it.
   Answer:
   - Converse: If you reach out and take the gift of grace, then it can be yours.
   - Inverse: If the gift of grace cannot be yours, then you'll not reach out and take it.
   - Contrapositive: If you do not reach out and take the gift of grace, then it cannot be yours.

2. Construct a complete truth table to help you determine if the following argument is valid or not. State whether it is valid or not, indicate the entries in the truth table that led you to your answer, and explain why those entries support your answer.

$$\begin{array}{rl} & \sim p \vee q \\ & r \rightarrow (\sim q) \\ \hline \therefore & p \rightarrow (\sim r) \end{array}$$

Answer:

p	q	r	$\sim p$	Premise $\sim p \vee q$	$\sim q$	Premise $r \rightarrow (\sim q)$	$\sim r$	Conclusion $p \rightarrow (\sim r)$	
1	1	1	0	1	0	0	0	0	
1	1	0	0	1	0	1	1	1	$\leftarrow$ Critial row
1	0	1	0	0	1	1	0	0	
1	0	0	0	0	1	1	1	1	
0	1	1	1	1	0	0	0	1	
0	1	0	1	1	0	1	1	1	$\leftarrow$ Critial row
0	0	1	1	1	1	1	0	1	$\leftarrow$ Critial row
0	0	0	1	1	1	1	1	1	$\leftarrow$ Critial row

The critical rows are the rows where all the premises are true. Since the conclusion is true for all critical rows, this argument is valid.

3. This question allows you to practice two different ways that will verify that the following two statements are logically equivalent.

   - $P \leftrightarrow Q$
   - $(Q \wedge P) \vee \sim (P \vee Q)$

   (a) Construct a Complete Truth Table to show that the following two statements above are logically equivalent (they are indeed logically equivalent).

   Answer:

P	Q	$(Q \wedge P)$	$P \vee Q$	$\sim (P \vee Q)$	$(Q \wedge P) \vee \sim (P \vee Q)$	$P \leftrightarrow Q$
1	1	1	1	0	1	1
1	0	0	1	0	0	0
0	1	0	1	0	0	0
0	0	0	0	1	1	1

   (b) Next use only the rules given in table 1.1.1 along with definitions of biconditional and conditional as presented on the reference sheet to show that they are logically equivalent. Use the format of the proof shown in class — each line of your proof must be justified with one of the rules from table 1.1.1 and you must tell which line that rule was applied to get the new line you are adding to your proof.

   Answer:

Line	Statement	Rule	Line
1	$(P \rightarrow Q) \wedge (Q \rightarrow P)$	Def. of Biconditional	given
2	$(\sim P \vee Q) \wedge (\sim Q \vee P)$	Def. of Cond.	1
3	$((\sim P \vee Q) \wedge \sim Q) \vee ((\sim P \vee Q) \wedge P)$	Distrib.	2
4	$((\sim P \wedge \sim Q) \vee (Q \wedge \sim Q)) \vee ((\sim P \wedge P) \vee (Q \wedge P))$	Distrib. & Comm	3
5	$(c \vee (Q \wedge P)) \vee ((\sim P \wedge \sim Q) \vee c)$	Comm & Negation	4
6	$(Q \wedge P) \vee (\sim P \wedge \sim Q)$	Identity	5
7	$(Q \wedge P) \vee \sim (P \vee Q)$	DeMorgan's	6

4. Use any of the rules you were given to complete the two proofs below. Use the same format as was shown in class for these proofs — each line of your proof must be justified with the rule and line numbers you used to obtain that line.

(a)		(b)	
P1	$(p \to q) \wedge (r \to s)$	P1	$p \to q$
P2	$(s \wedge q) \to (\sim v)$	P2	$\sim q \vee r$
P3	v	P3	$s \vee (v \wedge \sim r)$
$\therefore$	$(\sim p) \vee (\sim r)$	$\therefore$	$\sim s \to \sim (p \vee \sim v)$

Answer:

(a)

Line	Statement	Rule	Lines Used
1	$\sim (s \wedge q)$	Modus Tollens	P3,P2
2	$\|p \wedge r$	Assume	
3	$\|p$	Conjunctive Simplification	2
4	$\|p \to q$	Conjunctive Simplification	P1
5	$\|q$	Modus Ponens	4,3
6	$\|r$	Conjunctive Simplification	2
7	$\|r \to s$	Conjunctive Simplification	P1
8	$\|s$	Modus Ponens	7,6
9	$\|s \wedge q$	Conjunctive Addition	5,8
10	$\| \sim v$	Modus Ponens	P2,9
11	$\|v \wedge \sim v$	Conj. Add.	10,P3
12	$\sim (p \wedge r)$	Closing cond world w/contradiction	2-11
13	$(\sim p) \vee (\sim r)$	DeMorgan's Law	12

(b)

Line	Statement	Rule	Lines Used
1	$\| \sim s$	Assume	—
2	$\|v \wedge \sim r$	Disjunctive Syllogism	P3,1
3	$\| \sim r$	Conjunctive Simplification	2
4	$\| \sim q$	Disjunctive Syllogism	P2, 3
5	$\| \sim p$	Modus Tollens	P1,4
6	$\|v$	Conjunctive Simplification	2
7	$\| \sim p \wedge v$	Conjunctive Addition	5,6
8	$\| \sim (p \vee \sim v)$	DeMorgan's Law	7
9	$\sim s \to \sim (p \vee \sim v)$	Closing cond world	1-8