CMSC 250 Fall 2004 — Homework #5

Due Friday, Oct. 8 at the beginning of your LECTURE section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

Prove each of the following statements true or false. Remember, a counterexample may only be used to prove that a "for all" statement is false, and all counterexamples must include specific values and enough algebra/justification to show that they are truly counterexamples.
Your proofs must be complete as discussed in class -including things like a sequence of statements that are known to be true and the reason you know that the statement you wrote is true.

1. There is an integer n such that $2n^2 - 5n + 2$ is prime.
2. For all integers n, if n is prime then $(-1)^n = -1$.
3. For all integers $n \geq 1$, $n(n+1)(n+2)(n+3)$ is one less than a perfect square.
4. Given any two rational numbers r and s with $r < s$, there is another rational number between r and s. (Hint: consider $\frac{r+s}{2}$.)
5. The sum of any two even numbers is a multiple of 4.
6. For all integers a and b, if $a|b$ then $a^2|b^2$.
7. If n is an odd integer, then $n^4 \equiv_4 1$.