CMSC 250 Fall 2004 — Homework 6

Due Wed., Oct. 13 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Prove each of the following. Either give a complete formal proof that the statement is true or a specific counter example with justification to prove that it is false.

   (a) For any integer n,
   $$\left\lfloor \frac{n^2}{4} \right\rfloor = \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right).$$

   (b) For all real numbers x, $\lfloor x^2 \rfloor = \lfloor x \rfloor^2$.

   (c) For all real numbers x and y, if x is irrational and y is rational then $x - y$ is irrational.

   (d) For all integers a, b and c, if $a|b$ and $a \nmid c$ then $a \nmid (b+c)$.

   (e) $\log_5 2$ is irrational.

   (f) For all integers n, if $n > 2$ then there is a prime number p such that $n < p < n!$ (hint: consider $n! - 1$).