CMSC 250 Fall 2004 — Homework 7

Due Wed., Oct. 20 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Reduce each of the following expressions to a single numeric value:

   (a) $\displaystyle\sum_{j=1}^{5} \frac{(-1)^j}{j}$

   (b) $\displaystyle\prod_{k=0}^{10} \frac{10-k}{2^k}$

   (c) $\displaystyle\prod_{i=1}^{3} \left(\sum_{j=i}^{3} i \cdot j \right)$

2. For each of the following, change the sum and/or products into sum or product notation:

   (a) $\displaystyle\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!}$

   (b) $\displaystyle n + \frac{n-1}{2!} + \frac{n-2}{3!} + \frac{n-3}{4!} + \cdots + \frac{1}{n!}$

3. Prove the following facts for $n \geq 1$:

   (a) $11^n - 6$ is divisible by 5.

   (b) $\displaystyle\sum_{k=1}^{n} k(k-1)(k-2)(k-3) = \frac{(n+1)n(n-1)(n-2)(n-3)}{5}$

   (c) $\displaystyle\sum_{i=1}^{n} [i \cdot i!] = (n+1)! - 1$

4. A convex polygon is a polygon with all interior angles ≤ 180 degrees. Prove that the sum of the interior angles in any n-sided convex polygon is exactly $(n-2) \cdot 180$ degrees, for all $n \geq 3$. (Reminder: The sum of the interior angles of any triangle is 180 degrees.)