

CMSC 250 Fall 2004 — Homework 8

Due Wed., Oct. 27 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Prove the following for integer n :

(a) $\prod_{i=2}^n \left(1 - \frac{1}{i^2}\right) = \frac{n+1}{2n}$ for $n \geq 2$.

(b) $2^n < (n+1)!$ for $n \geq 2$.

(c) $\sqrt{n} < \sum_{i=1}^n \frac{1}{\sqrt{i}}$ for $n \geq 2$.

(d) Prove that any sum of two or more rational numbers is rational.

(e) Suppose $a_1, a_2, a_3, \dots$ is a sequence defined as follows:

$$\begin{aligned} a_1 &= 1, a_2 = 3 \\ a_k &= a_{k-2} + 2a_{k-1} \quad \text{for all integers } k \geq 3. \end{aligned}$$

Prove that a_n is odd for all $n \geq 1$.

(f) An L-tromino is composed by three squares and shaped like an L: . Use mathematical induction to prove that for any integer $n \geq 1$, if one square is removed from a $2^n \times 2^n$ checkerboard, the remaining squares can be completely covered by L-trominos.

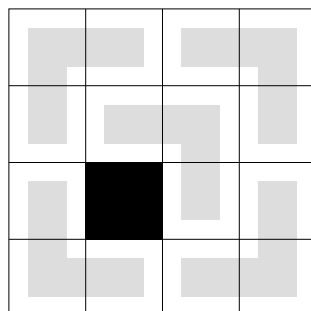


Figure 1: Example

2. Calculate the following: $1 - 2 + 2^2 - 2^3 + \dots + (-1)^n 2^n$ where $n > 0$.