

CMSC 250 Fall 2004 — Homework 14

Due Never at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. In the following, the relation R is an equivalent relation on the set A . Find the distinct equivalence classes of R .

- (a) $X = \{-1, 0, 1\}$ and $A = P(X)$. R is defined on $P(X)$ as follows: For all sets s and t in $P(X)$,

$$s R t \Leftrightarrow \text{the sum of the elements in } s \text{ equals the sum of the elements in } t$$

- (b) A is the set of all strings of length 2 in 0's, 1's, and 2's. R is defined on A as follows: For all strings s and t in A ,

$$s R t \Leftrightarrow \text{the sum of the characters in } s \text{ equals the sum of the characters in } t$$

2. Let P be the set of all points in the Cartesian plane except the origin. R is the relation defined as follows: For all p_1 and p_2 in P ,

$$p_1 R p_2 \Leftrightarrow p_1 \text{ and } p_2 \text{ lie on the same half-line emanating from the origin.}$$

Proof that the relation is an equivalence relation, and describe the distinct equivalence classes.

3. Let R be a binary relation on a set A and suppose R is symmetric and transitive. Prove the following: If for every x in A there is a y in A such that $x R y$, then R is an equivalence relation.
4. For each of the following either prove that it is a partial order relation or indicate why it is not a partial order relation.

- (a) Define a relation R on the set $\mathbf{Z}$ of all integers as follows: For all $m, n \in \mathbf{Z}$,

$$m R n \Leftrightarrow \text{every prime factor of } m \text{ is a prime factor of } n$$

- (b) Define a relation R on the set $\mathbf{R}$ of all real numbers as follows: For all $x, y \in \mathbf{R}$,

$$x R y \Leftrightarrow x^2 \leq y^2$$

5. Let $A = \{a, b, c, d\}$, and let R be the relation

$$R = \{(a, a), (b, b), (c, c), (d, d), (c, b), (a, d), (b, a), (b, d), (c, d), (c, a)\}$$

Is R a total order on A ? justify your answer.

6. Define each of the following as it pertains to a graph

- (a) Path
- (b) Simple Circuit
- (c) Circuit
- (d) Euler Circuit
- (e) Hamiltonian Circuit