1. Prove that the following are logically equivalent pairs - use both the truth table method and the formal proof method to get extra practice.

a.	$p \vee q$	$(p \rightarrow q) \rightarrow q$
b.	$\sim (p \leftrightarrow q)$	$\sim p \leftrightarrow q$

ANSWERS:

p	q	$p \vee q$	$p \rightarrow q$	$(p \rightarrow q) \rightarrow q$
1	1	1	1	1
1	0	1	0	1
0	1	1	1	1
0	0	0	1	0

1	$(p \rightarrow q) \rightarrow q$	RHS
2	$\sim (p \rightarrow q) \vee q$	def of impl
3	$\sim (\sim p \vee q) \vee q$	def of impl
4	$(p \wedge \sim q) \vee q$	DM
5	$(p \vee q) \wedge (\sim q \vee q)$	Distrib
6	$(p \vee q) \wedge t$	Negation
7	$p \vee q$	identity

p	q	$\sim p$	$p \leftrightarrow q$	$\sim (p \leftrightarrow q)$	$\sim p \leftrightarrow q$
1	1	0	1	0	0
1	0	0	0	1	1
0	1	1	0	1	1
0	0	1	1	0	0

1	$\sim (p \leftrightarrow q)$	LHS
2	$\sim [(p \rightarrow q) \wedge (q \rightarrow p)]$	def of bicond
3	$\sim (p \rightarrow q) \vee \sim (q \rightarrow p)$	DM
4	$\sim (\sim p \vee q) \vee \sim (\sim q \vee p)$	def of impl
5	$(p \wedge \sim q) \vee (q \wedge \sim p)$	DM and DN
6	$((p \wedge \sim q) \vee q) \wedge ((p \wedge \sim q) \vee \sim p)$	Dist
7	$((p \vee q) \wedge (\sim q \vee q)) \wedge ((p \vee \sim p) \wedge (\sim q \vee \sim p))$	Dist
8	$((p \vee q) \wedge t) \wedge (t \wedge (\sim q \vee \sim p))$	negation
9	$(p \vee q) \wedge (\sim q \vee \sim p)$	identity
10	$(\sim p \rightarrow q) \wedge (q \rightarrow \sim p)$	Def of impl
11	$\sim p \leftrightarrow q$	def of bicond

2. Albert, Bill and Casey are all suspects in a police investigation. Each of them make the following statements.

   - Albert: Bill is guilty and Casey is innocent.
   - Bill: If Albert is guilty, then so is Casey.
   - Casey: I am innocent, but at least one of the others is guilty.

   Let a, b and c be "Alber is innocent", "Bill is innocent" and "Casey is innocent" respectively. And do each of the following exercises.

   a. Rewrite the suspects' statements in terms of a,b, and c with our regular logic operators.
   ANSWER:
      - Albert: $\sim b \wedge c$
      - Bill: $\sim a \rightarrow \sim c$
      - Casey: $c \wedge (\sim a \vee \sim b)$

b. If all are innocent, who committed perjury (lied in their official statement)?

ANSWER: If all are innocent, Albert and Casey both must be lying because they both say that one other person is guilty. Bill is telling the truth because he said nothing about anyone being guilty only an implication based on someone else's guilt.

c. If all the statements are true, who is innocent?

ANSWER: If all statements are true, Albert and Casey are innocent and Bill is Guilty.

d. If the innocent were truthful and the guilty lied, who is innocent?

ANSWER: Bill is innocent and honest and Albert and Casey are guilty and dishonest.

3. For each of the following either give a formal proof to show that the argument is valid or give values for the variables to demonstrate a counter example to show that it is not a valid argument form.

a.

$a \vee (b \wedge c)$
$a \rightarrow c$
c

ANSWER:

1	$(a \vee b) \wedge (a \vee c)$	Distrib	P1
2	$a \vee c$	Conj Simp	1
3	$\sim a \vee c$	Def of Impl	P2
4	$(a \vee c) \wedge (\sim a \vee c)$	Conj Add	2,3
5	$(a \wedge \sim a) \vee c$	Distrib and Comm	4
6	$contradiction \vee c$	Negation	5
7	c	identity	6

ALTERNATIVE ANSWER:

1	$(a \vee b) \wedge (a \vee c)$	distrib	P1
2	$(a \vee c)$	Conj Simp	1
3	$\mid \sim c$	Assume	
4	$\mid a$	disj syll	2,3
5	$\mid \sim a$	MT	P2,3
6	$\mid a \wedge \sim a$	Conj Add	4,5
7	c	CCW w/contra	3-6

b.

$(d \vee e) \rightarrow (f \rightarrow g)$
$(\sim g \vee h) \rightarrow (d \wedge f)$
g

ANSWER:

1	$\mid \sim g$	Assume	
2	$\mid \sim g \vee h$	Disj Add	1
3	$\mid d \wedge f$	MP	2,P2
4	$\mid d$	Conj Simp	3
5	$\mid d \vee e$	Disj Add	4
6	$\mid f \rightarrow g$	MP	P1,5
7	$\mid \sim f$	MT	6,1
8	$\mid f$	Conj Simp	3
9	$\mid f \wedge \sim f$	conj add	7,8
10	g	CCW w/contra and DN	1-9

c.

$(h \rightarrow i) \wedge (j \rightarrow k)$
$(i \vee k) \rightarrow m$
$\sim m$
$\sim (h \vee j)$

ANSWER:

1	$\sim (i \vee k)$	MT	P2, P3
2	$\sim i \wedge \sim k$	DM	1
3	$\sim i$	Conj Simp	2
4	$\sim k$	Conj Simp	2
5	$h \rightarrow i$	Conj Simp	P1
6	$\sim h$	MT	3,5
7	$j \rightarrow k$	Conj Simp	P1
8	$\sim j$	MT	7,4
9	$\sim h \wedge \sim j$	Conj Add	6,8
10	$\sim (h \vee j)$	DM	9

d.

$(m \vee n) \rightarrow (d \wedge p)$
$(d \vee q) \rightarrow (\sim r \wedge s)$
$(r \vee t) \rightarrow (m \wedge u)$
$\sim r$

ANSWER:

1	$\|r$	Assume	
2	$\|r \vee t$	Disj Add	1
3	$\|m \wedge u$	MP	2,p3
4	$\|m$	Conj Simp	3
5	$\|m \vee n$	Disj Add	4
6	$\|d \wedge p$	MP	P1,5
7	$\|d$	Conj Simp	6
8	$\|d \vee q$	Disj Add	7
9	$\| \sim r \wedge s$	MP	8,P2
10	$\| \sim r$	Conj Simp	9
11	$\|r \wedge \sim r$	Conj Add	1,10
12	$\sim r$	CCW w/contra	1-11

e.

$(k \rightarrow \sim w) \wedge (x \rightarrow y)$
$(\sim w \rightarrow z) \wedge (y \rightarrow \sim a)$
$(z \rightarrow \sim b) \wedge (\sim a \rightarrow g)$
$k \wedge x$
$\sim b \wedge g$

ANSWER:

1	$z \rightarrow \sim b$	Conj Simp	P3
2	$\sim w \rightarrow z$	Conj Simp	P2
3	$\sim w \rightarrow \sim b$	Hypo Syll	1,2
4	$k \rightarrow \sim w$	Conj Simp	P1
5	$k \rightarrow \sim b$	Hypo Syll	3,4
6	k	Conj Simp	P4
7	$\sim b$	MP	5,6
8	x	Comj Simp	P4
9	$x \rightarrow y$	Conj Simp	P1
10	y	MP	8,9
11	$y \rightarrow \sim a$	Conj Simp	P2
12	$\sim a$	MP	10,11
13	$\sim a \rightarrow g$	Conj Simp	P3
14	g	MP	12,13
15	$\sim b \wedge g$	Conj Add	7,14