CMSC 250 Quiz #12 ANSWERS Wed., Nov. 17, 2004

Write all answers legibly in the space provided. The number of points possible for each question is indicated in square brackets – the total number of points on the quiz is 30, and you will have exactly 15 minutes to complete this quiz. You may not use calculators, textbooks or any other aids during this quiz.

Note: you only need to take the answer as far as a mathematical expression including addition, subtraction, multiplication, division, exponents and factorials.

1. [3 pnts.] How many solutions are there to the equation $n_1 + n_2 + n_3 + n_4 = 23$ such that $\forall i \in Z, n_i \in Z^+$?

$$\binom{19 + (4-1)}{19} = \frac{22!}{19!3!}$$

2. [3 pnts.] How many solutions are there to the equation $n_1 + n_2 + n_3 + n_4 = 23$ such that $\forall i \in Z, (n_i \in Z^+ \wedge n_i \geq (i+1))$?

$$\binom{9 + (4-1)}{9} = \frac{12!}{9!3!}$$

3. [3 pnts.] Assume there are 20 students in a class. The teacher is going to give 6 A's, 2 B's, 8 C's, 2 D's and the rest are F's. How many ways can those grades be assigned to the 20 students?

$$\binom{20}{6}\binom{14}{2}\binom{12}{8}\binom{4}{2}\binom{2}{2} = \frac{20!}{6!2!8!2!2!}$$

4. [3 pnts.] Assume there are 20 students in the class half of which are boys and half girls. The teacher is going to give 6 A's, 2 B's, 8 C's, 2 D's and the rest are F's (same as the previous problem), but now he has the added restriction that the number of girls receiving any letter grade must be the same as the number of boys receiving that same letter grade. How many ways can those grades be assigned to the 20 students?.

$$(\binom{10}{3}\binom{7}{1}\binom{6}{4}\binom{2}{1}\binom{1}{1})^2 = \frac{10!}{3!4!} \times \frac{10!}{3!4!}$$

5. [3 pnts.] In the expansion of $(a+b)^{20}$, what is the coefficient on the term $a^7b^{13}$?

$$\binom{20}{7} = \frac{20!}{7!13!}$$

6. [15 pnts.] This question will deal with drawing balls out of a bag in such a way that you can not see what the color of the ball is before you draw it out. The balls are all the same size and indistinguishable by feel. There are 7 blue balls, 12 red balls and 9 yellow balls. Note after each part of this question, the balls drawn are put back into the urn and the restrictions of one part of this question do not affect any of the others.

   a. Assume in addition to their color, each ball also has a number on it, and each number is unique (from 1 to 28). You draw one ball at a time out of the bag until you have drawn 10 balls. You place them in a line on the table in the order they were drawn. How many different sequences of balls could you possibly have?

$$P(28,10) = \frac{28!}{(28-10)!} = \frac{28!}{18!}$$

   b. Assume you draw all of the balls out of the bag and place them in a single line on the table. How many different orders could you have (Notice that the numbers that were on the balls in the previous part of this question - are not there for this part of the question.)

$$\frac{28!}{7!12!9!}$$

   c. Assume you draw five balls from the bag, what is the probability that they will all be the same color?

$$\frac{\binom{7}{5}+\binom{12}{5}+\binom{9}{5}}{\binom{28}{5}} = \frac{\frac{7!}{5!2!}+\frac{12!}{5!7!}+\frac{9!}{5!4!}}{\frac{28!}{5!23!}}$$

   d. Assume you draw three balls from the bag, what is the probability that you will have one of each color?

$$\frac{\binom{7}{1}\binom{12}{1}\binom{9}{1}}{\binom{28}{3}} = \frac{7\times 12\times 9}{\frac{28!}{3!25!}}$$

   e. Assume you take one ball out write down it's color and then replace it back into the bag before drawing again. What is the probability that the second ball drawn will be the same color as the first ball was?

$$\left(\frac{7}{28}\right)^2+\left(\frac{12}{28}\right)^2+\left(\frac{9}{28}\right)^2$$