Name (PRINTED): _____

Student ID #: _____

Section # (or TA's: name and time) _____

# CMSC 250 Quiz #8 ANSWERS Wednesday, Oct. 20, 2004

Write all answers legibly in the space provided. The number of points possible for each question is indicated in square brackets – the total number of points on the quiz is 30, and you will have exactly 15 minutes to complete this quiz. You may not use calculators, textbooks or any other aids during this quiz.

1. [12 pnts.] Expand each of the following – if there are more than 5 terms, you only need to tell the first five terms of each.

   a. $\sum_{i=4}^{7} \frac{i^2}{2}$

   $\frac{16}{2} + \frac{25}{2} + \frac{36}{2} + \frac{49}{2}$

   b. $\forall k > 1 \;\; a_k = 2^{k+1}$

   $a_2 = 2^3 = 8$
   $a_3 = 2^4 = 16$
   $a_4 = 2^5 = 32$
   $a_5 = 2^6 = 64$
   $a_6 = a^7 = 128$

   c. $\prod_{m=2}^{10}(m+1)$

   $3 \times 4 \times 5 \times 6 \times 7$

2. [8 pnts.] Find the numeric value (or a single fraction) for each of the following

   a. $\sum_{i=3}^{5} \frac{i-1}{i}$

   $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} = \frac{40}{60} + \frac{45}{60} + \frac{48}{60} = \frac{133}{60}$

   b. $\prod_{j=2}^{4}(j^2+1)$

   $5 \times 10 \times 17 = 850$

3. [pnts.] Use induction to prove the following statement.

$$\forall n \in Z^+ \ \sum_{k=1}^{n}(3k-2) = \frac{n(3n-1)}{2}$$

**Base Case:** (n=1)
$\sum_{k=1}^{1}(3k-2) = 3(1) - 2 = 1$
$\frac{1(3(1)-1)}{2} = \frac{1(2)}{2} = 1$

**Inductive Hypothesis:**(n=x)
$\sum_{k=1}^{x}(3k-2) = \frac{x(3x-1)}{2}$

**Inductive Step:**(n=x+1)
show: $\sum_{k=1}^{x+1}(3k-2) = \frac{(x+1)(3(x+1)-1)}{2}$
Or after a little algebra: $\sum_{k=1}^{x+1}(3k-2) = \frac{(x+1)(3x+2)}{2}$

proof:
$\sum_{k=1}^{x+1}(3k-2) = \sum_{k=1}^{x}(3x-2) + 3(x+1) - 2$
$= \frac{x(3x-1)}{2} + 3(x+1) - 2$ by the Inductive Hypothesis
$= \frac{3x^2-x}{2} + 3x + 1$
$= \frac{3x^2-x+6x+2}{2} = \frac{(x+1)(3x+2)}{2}$
QED