

Graphs

- Formal Definition of Graph G is 2 finite sets
 - $V(G)$ = set of vertices & $E(G)$ = set of edges
- Example:
 - $V(H) = \{a,b,c,d,e\}$ $E(H) = \{\{a,c\},\{c,e\},\{e,b\},\{b,d\},\{d,a\}\}$
 - $V(K) = \{a,b,c,d\}$ $E(K) = \{(a,b),(b,a),(a,d),(d,a),(c,c)\}$
- Variations
 - digraph : edges are ordered tuples
 - multi-graph : edge list is a multiset (bag) not set
 - simple graph : no parallel edges and no “reflexive loops”
 - connected graph: can get from any vertex to any other
 - complete graph: has an edge for every pair of vertices
 - complete bipartite graph: 2 subsets of vertices (u and v), edge from each v to each u , no edges connecting u elements and no edges connecting v elements
- Subgraph: H is a subgraph of $G \leftrightarrow V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$

Counting in Graphs

- Number of Edges Possible
 - complete (simple) graph $n(E(K_x)) = \sum_{i=1}^{x-1} i$
 - complete bipartite graph $n(E(K_{x,y})) = x * y$
- Degree of a Vertex
 - = number of times that vertex is the endpoint of an edge
 - = number of edges incident on it with self-loops counted twice

Isomorphism

- $(G \text{ is isomorphic to } H) \leftrightarrow$ There exists a bijective function $f_1: V(G) \rightarrow V(H)$ and a bijective function $f_2: E(G) \rightarrow E(H)$

Traversing a Graph

Name	Repeated Edges	Repeated Vertices	Same end/start
Walk	allowed	allowed	allowed
Path	NO	allowed	allowed
Simple Path	NO	NO	NO
Closed Walk	allowed	allowed	YES
Circuit	NO	allowed	YES
Simple Circuit	NO	only the start/end	YES

Euler Circuit

- A circuit that contains every edge and every vertex
 - starts and stops at the same point
 - uses every vertex at least once
 - uses every edge exactly once
- G has an Euler Circuit $\leftrightarrow$ G is a connected graph and
Every vertex of G has even degree

Hamiltonian Circuit

- A simple circuit that contains every vertex.
 - starts and stops at the same point
 - uses every vertex exactly once (except the first and last)
 - does not repeat an edge