CMSC421 Fall 2005 Homework 2

October 17, 2005

1 *Problem 1: Constraint Satisfaction (25 pts)*

a. (i) See Figure 1.

(ii) 1=B, 2=G, 3=R, 4=R, 5=B.

(iii) Count number of nodes expanded: 8.

b. (i) See Table 1.

(ii) 1=B, 2=G, 3=R, 4=R, 5=B

(iii) 5.

c. (i) $3^5 = 243$.

(ii) Take the circle formed by region 1,2,4. As they are mutually connected to each other, each region has to take one unique color. The second circle formed by region 2,4,5 then determine the color of region 5. Region 2 and 5 determines the color of region 3. Therefore, $3! = 6$.

2 *Problem 2: Game Playing (15 pts)*

a. See Figure 2.

b. (i) All leaf nodes are terminal nodes.

(ii) See Figure 2.

(iii) From left to right, number the root's children as node 1,2,3,4. Those nodes in the subtrees rooted at node 2,3,4 (including node 2,3,4) will be pruned.

Table 1: Forward Checking

	1	2	3	4	5
	BGR	BGR	BGR	BGR	BGR
1 = B	B	GR	BGR	GR	BGR
2 = G	B	G	BR	R	BR
4 = R	B	G	BR	R	B
5 = B	B	G	R	R	B
3 = R	B	G	R	R	B

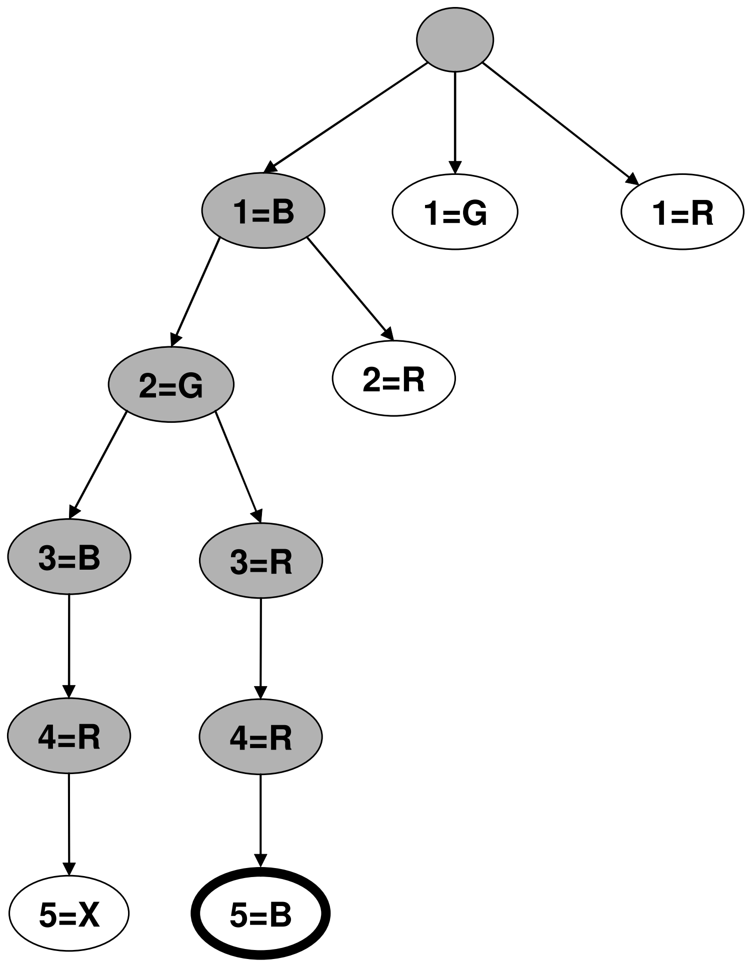

Figure 1: Backtracking Tree

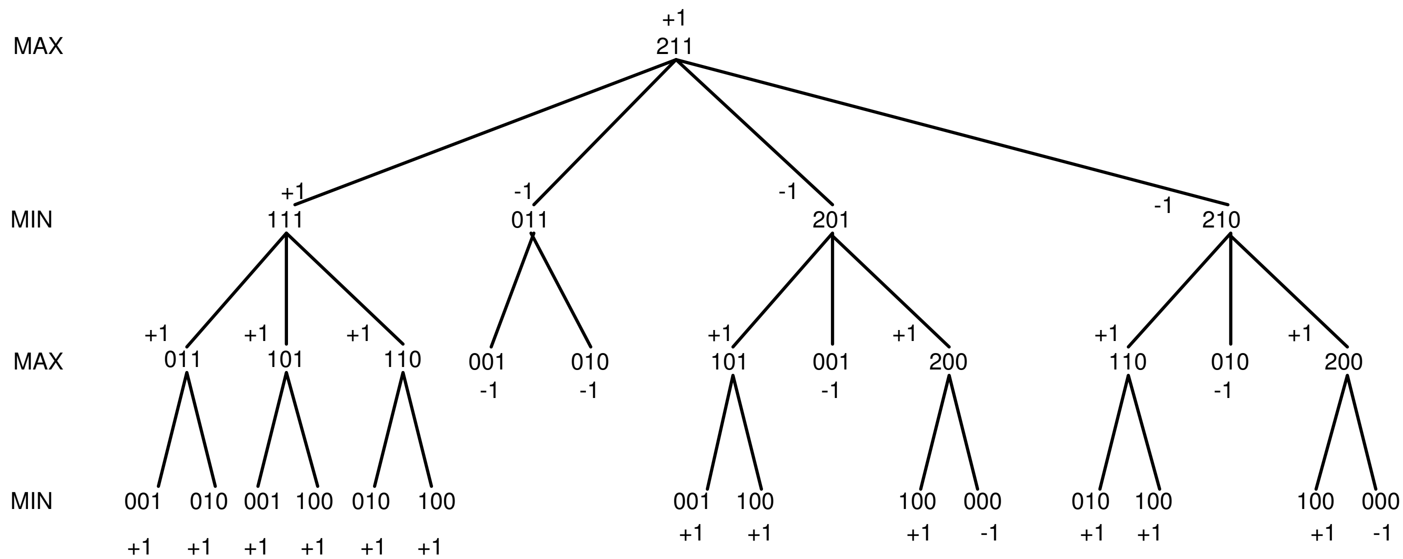

Figure 2: Game Tree of Nim211

3 *Problem 3: Propositional Logic - Models (9 pts)*

These can be computed by counting the rows in a truth table that come out true. Remember to count the propositions that are not mentioned; if a sentence mentions only A and B, then we multiply the number of models for $\{A,B\}$ by 2^2 to account for C and D.

a. 6

b. 12

c. 8. The correct interpretation is $A \Leftrightarrow (B \Leftrightarrow C)$, therefore the number of models is 4×2. However, if you interpretted it as $(A \Leftrightarrow B) \Leftrightarrow C$ or were explicit about this, you also got credit.

4 *Problem 4: Propositional Logic - Validity(16 pts)*

We could use truth table or applying equivalence rules to determine validity of a statement.

a. Valid since $Smoke \Rightarrow Smoke \equiv \sim Smoke \vee Smoke$ is a tautology.

b. Neither (or Satisfiable) by examining the truth table 2.

c. Neither (or Satisfiable).

d. Valid.

e. Valid.

f. Valid.

g. Valid.

Table 2: Truth Table of $Smoke \Rightarrow Fire$

$Smoke$	$Fire$	$Smoke \Rightarrow Fire$
T	T	T
T	F	F
F	T	T
F	F	T

h. Neither (or Satisfiable).

5 *Problem 5: Propositional Logic - Resolution (30 pts)*

From the first two statements, we see that if it is mythical, then it is immortal; otherwise it is a mammal. So it must be either immortal or a mammal, and thus horned. That means it is also magical. However, we can't deduce anything about whether it is mythical.

First, we introduce some symbols:
Y: Unicorn is mYthical
I: Unicorn is Immortal
M: Unicorn is a Mammal
H: Unicorn is Horned
G: Unicorn is maGical
Therefore, the propositional knowledgebase will be

$$\begin{aligned} Y &\Rightarrow I \\ \sim Y &\Rightarrow M \\ I \vee M &\Rightarrow H \\ H &\Rightarrow G \end{aligned}$$

The final knowledgebase will be as following after applying Implication Elimination, DeMorgan's and Distributivity logical equivalences laws:

1. $\sim Y \vee I$
2. $Y \vee M$
3. $\sim I \vee H$
4. $\sim M \vee H$
5. $\sim H \vee G$

Now, we'd like to know if Y? G? H?

Using Resolution Refutation, we introduce $\sim Y$ into the knowledgebase and hope that we can get false in order to prove Y. However, it doesn't produce false (Note that we cann't say Q is not entailed by the knowledgebase). Hence, we could infer nothing on mythical.

Include $\sim G$ into the knowledgebase.

1. $\sim Y \vee I$
2. $Y \vee M$
3. $\sim I \vee H$
4. $\sim M \vee H$
5. $\sim H \vee G$
6. $\sim G$ *negated goal*
7. $\sim H$ *5, 6*
8. $\sim I$ *3, 7*
9. $\sim Y$ *1, 8*
10. M *2, 9*
11. H *4, 10*
12. $false$ *7, 11*

We can prove that unicorn is magical. Similarly, we can prove unicorn is horned.

6 *Problem 6: First Order Logic - Representation (25 pts)*

There are different ways of doing this.

Let the basic vocabulary be as follows:
Takes(x,c,s): student x takes course c in semester s;
Passes(x,c,s): student x passes course c in semester s;
Score(x,c,s): the score obtained by student x in course c in semester s;
$x > y$: x is greater than y.
F and G: specific French and Greek courses;
Buys(x,y,z): x buys y from z;
Sells(x,y,z): x selss y to z;
Shaves(x,y): person x shaves person y;
Born(x,c): person x is born in country c;
Parent(x,y): x is a parent of y;
Citizen(x,c,r): x is a citizen of country c for reason r;

Resident(x,c): x is a resident of country c;
Fools(x,y,t): person x folls person y at time t;
$Student(x)$,$Person(x)$, $Man(x)$,$Barber(x)$,$Expensive(x)$,$Agent(x)$,$Insured(x)$,$Smart(x)$,$Politician(x)$: predicates satisfied by members of the corresponding categories.

a. Some students took French in spring 2001.
$\exists x\ Student(x) \wedge Takes(x, F, Spring2001)$.

b. Every student who takes Frnech passes it.
$\forall x, s\ Student(x) \wedge Takes(x, F, s) \Rightarrow Passes(x, F, s)$.

c. Only one student took Greek in spring 2001.
$\exists x\ Student(x) \wedge Takes(x, G, Spring2001) \wedge \forall y\ y \neq x \Rightarrow \sim Takes(y, G, Spring2001)$.

d. The best score in Greek is always higher than the best score in French.
$\forall s\ \exists x\ \forall y\ Score(x, G, s) > score(y, F, s)$.

e. Every person who buys a policy is smart.
$\forall x\ Person(x) \wedge (\exists y, z\ Policy(y) \wedge Buys(x, y, z)) \Rightarrow Smart(x)$.

f. No person buys an expensive policy.
$\forall x, y, z\ Person(x) \wedge Policy(y) \wedge Expensive(y) \Rightarrow \sim Buys(x, y, z)$.

g. There is an angetn who sells policies only to people what are not insured.
$\exists x\ Agent(x) \wedge \forall y, z\ \ Policy(y) \wedge Sells(x, y, z) \Rightarrow (Person(z) \wedge \sim Insured(z))$.

h. There is a barber who shaves all men in town who do not shave themselves.
$\exists x\ Barber(x) \wedge \forall y\ Man(y) \wedge \sim Shaves(y, y) \Rightarrow Shaves(x, y))$.

i. A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.
$\forall x\ Person(x) \wedge Born(x, UK) \wedge (\forall y\ Parent(y, x) \Rightarrow ((\exists r\ Citizen(y, UK, r)) \vee Resident(y, UK))) \Rightarrow Citizen(x, UK, Birth)$.

j. A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.
$\forall x\ Person(x) \wedge \sim Born(x, UK) \wedge (\exists y\ Parent(y, x) \wedge Citizen(y, UK, Birth)) \Rightarrow Citizen(x, UK, Descent)$.

k. Politicians can fool some of the people all of the time, and they can foll all of the people some of the time, but they can't fool all of the poeple all of the time.
$\forall x\ Politican(x) \Rightarrow$
$(\exists y\ \forall t\ Person(y) \wedge Fools(x, y, t)) \wedge$
$(\exists t\ \forall y\ Person(y) \Rightarrow Fools(x, y, t)) \wedge$

$\sim (\forall t\ \forall y\ Person(y) \Rightarrow Fools(x,y,t)).$