

Questions?

- Projects
- Homework

Two-way ANOVA

- Setting
 - Full factorial design: k columns, l rows
 - For each *cell* (treatment) you gathered from a random samples
 - *Mean*
 - *Standard deviation*
- Null hypothesis
 - F ratios (row column and interaction) follow a F distribution
 - *Main effect: F_{col} and F_{row}*
 - *Interaction: F_{int}*

F ratios

$$F_{col} = \frac{MS_{col}}{MS_{cell}}, \text{ with } \begin{cases} MS_{col} = \frac{SS_{col}}{df_{col}}, & SS_{col} = n_{col} \sum_{j=1}^k (\bar{X}_j - \bar{X}_{col})^2 \\ df_{col} = col_count - 1 \end{cases}$$

$$F_{row} = \frac{MS_{row}}{MS_{cell}}, \text{ with } \begin{cases} MS_{row} = \frac{SS_{row}}{df_{row}}, & SS_{row} = n_{row} \sum_{j=1}^l (\bar{X}_j - \bar{X}_{row})^2 \\ df_{row} = row_count - 1 \end{cases}$$

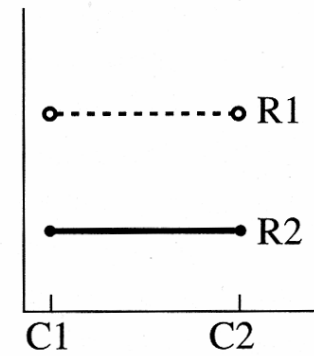
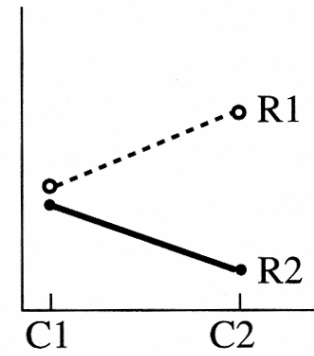
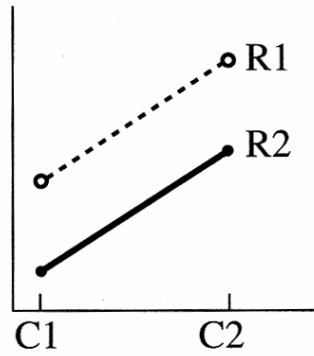
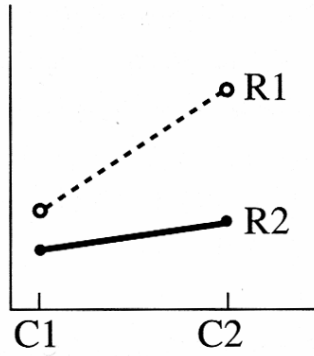
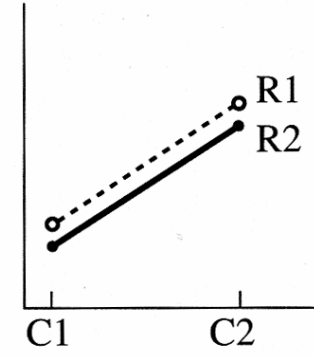
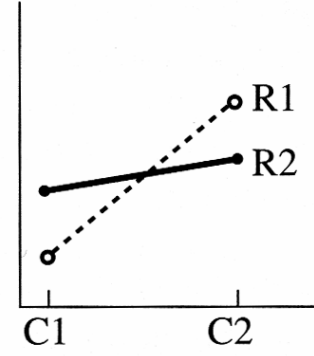
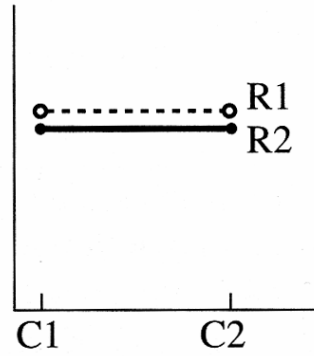
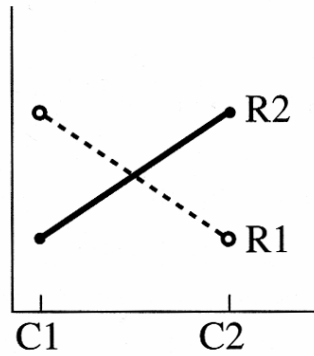
$$F_{int} = \frac{MS_{int}}{MS_{cell}}, \text{ with } \begin{cases} MS_{int} = \frac{SS_{int}}{df_{int}}, & SS_{int} = \underbrace{\left(\sum_{j=1}^{k \cdot l} \frac{T_j^2}{n_j} - \frac{T^2}{N} \right)}_{SS_{bet}} - SS_{row} - SS_{col}, \\ df_{int} = df_{row} \times df_{col} \end{cases}$$

$$\text{and } \begin{cases} MS_{cell} = \frac{SS_{cell}}{cell_count}, & SS_{cell} = \sum \sigma_j^2, \text{ assuming equal size cells} \\ df_{cell} = N_T - cell_count \end{cases}$$

Interpretation (General Linear Model)

$$\begin{aligned}\text{Score} = & \text{Grand mean} \\ & + \text{Row effect} \\ & + \text{Column effect} \\ & + \text{Interaction effect} \\ & + \text{Error}\end{aligned}$$

Interactions



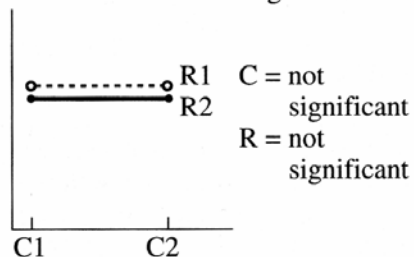
From Explaining Psychological Statistics (Cohen)

Interactions (Results)

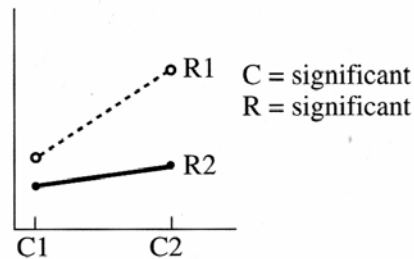
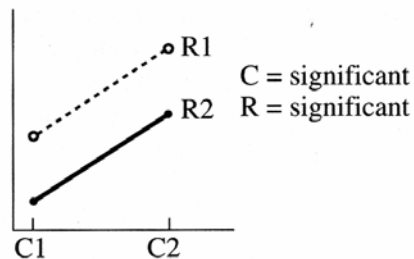
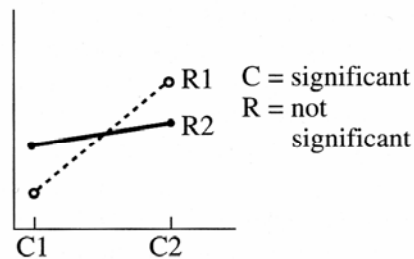
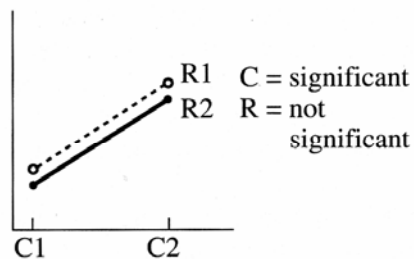
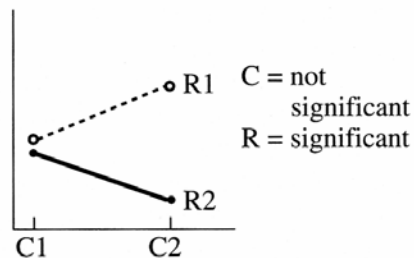
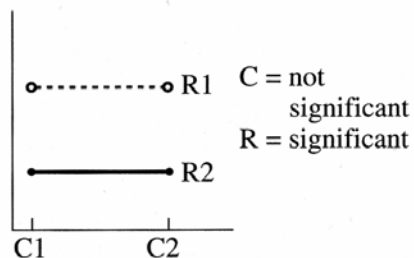
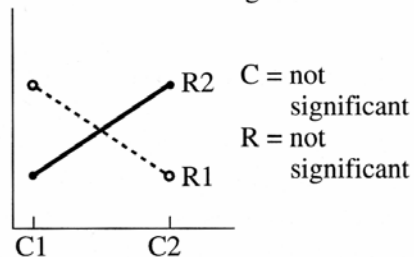
C = Column means

R = Row means

Interaction is not significant



Interaction is significant



Interactions (Caveats)

- If the interaction is significant
 - Main effect must be interpreted with care
- If the interaction is not significant
 - Main effect can be interpreted with 2 separate ANOVAs

Two-way ANOVA example

Other tests

- Repeated measure ANOVA
 - For within subject design
- Mixed Design ANOVA
 - For design which are both between and within subjects