Fall 2006 **CMSC 451: Homework 1** William Gasarch

Due Sep 13, at Beginning of class

1. (0 points). Write your name clearly. Staple your HW. READ Chap 4.1-4.4.
2. (25 points) For the following coin sets say whether they are greedy or not. If it is NOT greedy then give a counterexample.
   (a) 1,5,8
   (b) 1,5,9
   (c) 1,5,10
   (d) 1,5,11
   (e) Make a conjecture of the form "$(1, 5, v)$ is a greedy coin set iff XXX(v)." (no proof required)
3. (25 points) Find a function $f(d)$ such that the following is true and prove your result.

   IF for all $n \leq f(d)$ greedy is optimal for making change of n cents with coins $(1, 10, d)$ cents THEN $(1, 10, d)$ is greedy.
4. (25 points) (No proof required)
   (a) Find an infinite number of $(1, a, b)$ that ARE NOT a greedy coin set.
   (b) Find an infinite number of $(1, a, b)$ that ARE a greedy coin set.
5. (25 points) Let x be an instance of the INTERVAL SCHECULING problem (so x is a set of ordered pairs of start times and finish times of a job). Let $OPT(x)$ be the optimal number of jobs from x that can be scheduled. If A is an algorithm for this problem (which might not give the best result) then let $A(x)$ be the number of jobs that A would schedule. Below we present several BAD greedy algorithms for INTERVAL SCHEDULING. For each such algorithm find an instance x and a large number c such that $OPT(x) \geq cA(x)$. Try to make c as large as possible. Present both $OPT(x)$ and $A(x)$. Note that x has to be an actual set of ordered pairs of start and finish times.
   (a) Schedule by order of which ones START first.
   (b) Schedule by order of which jobs have the SHORTEST length.
   (c) Schedule by which one has the least other jobs competing with it.
6. (Extra Credit) Which coin sets $(1, 2, 3, \ldots, k)$ are Greedy and which are not? Prove your answer. (Hand in on Separate sheet- Dr. Gasarch grades this one.)