Fall 2006 **CMSC 451: Homework 2** William Gasarch

Due Sep 20, at Beginning of class

1. (0 points). Write your name clearly. Staple your HW. READ Chap 4.1-4.4.

2. (25 points) Recall that an instance of the INTERVAL SCHECULING problem is a set of ordered pairs of start times and finish times of a job. (NOTE- in an instance of INTERVAL SCHEDULING you cannot have the same interval twice. That is $\{[1,2],[1,2]\}$ would not be allowed.)

   (a) Find an instance of INTERVAL SCHEDULING that has exactly TWO optimal solutions. The instance must have at least 10 intervals.

   (b) Find an instance of INTERVAL SCHEDULING that has exactly THREE optimal solutions. The instance must have at least 10 intervals.

   (c) Let $k \geq 3$. Find an instance of INTERVAL SCHEDULING that has exactly k optimal solutions. The instance must have at least 10 intervals.

3. (25 points) Find a function f such that the following theorem is true (and prove it).

   If the maximum degree of a graph G is d then G is $f(d)$-colorable.

4. (25 points) The version of Dijistra's algorithm we presented in class took as input a graph G and output the numbers $d(v)$, the cost of going from s to v. Present an algorithm that also produces the PATH you would take going from s to v.

5. (25 points) Imagine that G is a weighted graph where we allow negative weights. In such a graph the very question of 'find the shortest path from s to v' may not make sense since a negative cycle may make it possible to go from s to v with lower and lower cost (negative numbers). However, one can still RUN Dijistra's algorithm on G. Where does the proof that Dijistra's algorithm gives the optimal cost break down?