Fall 2006 **CMSC 451: Homework 4** William Gasarch

Due Oct 4, at Beginning of class

1. (10 points). Write your name clearly. Staple your HW. READ Chap 4 on MST, clustering, Huffman Coding

2. (30 points) Give an example of a weighted graph on 10 nodes that has exactly 2 minimal spanning trees. Give an example of a weighted graph on 10 nodes that has exactly 3 minimal spanning trees. For any k show how to construct a graph (can be on any number of nodes) that has exactly k minimal spanning trees.

3. (30 points) You are given a weighted graph with the edges already sorted. You are also given k. Give a algorithm to find the best clustering of the graph into k parts. (You can assume that there is a data structure that does UNION in $O(1)$ steps and FIND in $O(\log^* n)$ steps.) Analyse the running time.

4. (30 points) Find an algorithm for the Clustering problem (input is $G = (V, E, w)$ and k) which is better than Kruskal's algorithm (read the rest of the problem to understand this). Analyse the run time of your algorithm. The run time will be of the form
$$O(V + E \log V + f(k))$$
where you need to specify f.