Fall 2006 **CMSC 451: Homework 6** William Gasarch

Due Friday Oct 27, at Beginning of class

(NOTE- HW is due on a Friday instead of a Wednesday.)

1. (0 points). Write your name clearly. Staple your HW. READ Chap 4 and the notes on BFS and DFS we will be giving out and posting on the website.

2. (30 points) Dr. Ydakra has come up with a way to multiply two n-bits numbers by doing 6 pairs of multiplying pairs of numbers that are $n/5$ bits long (you can ignore floor and ceiling issues) and 1000 additions of such numbers. Use this to obtain a divide-and-conquer algorithm for MULTIPLICATION. Express its run time in O-of notation. Is it better than the algorithm done in class and in the book (Page 232-233)?

3. (40 points) Let Z_{13} be the integers mod 13. Note that $1, 5, 8$, and 12 are 4th roots of unity mod 13:

   $1^4 = 1 \equiv 1 \pmod{13}$.

   $5^4 = 25 \times 25 \equiv -1 \times -1 \equiv 1 \pmod{13}$.

   $8^4 = 64 \times 64 \equiv -1 \times -1 \equiv 1 \pmod{13}$.

   $12^4 \equiv (-1)^4 \equiv 1 \pmod{13}$.

   Also note that 1 and 12 are the only 2nd roots of unity. (You can use $12 \equiv -1$ for easier calculations.)

   (a) Let $p(x) = 2x^3 + 5x^2 + 2x + 3$.

      i. Evaluate ($p(1)$ mod 13) and ($p(12)$ mod 13). (You can do this anyway you want, no need to use anything fancy.)

      ii. Evaluate $p(1), p(5), p(8), p(12)$ using the values $p(1)$ and $p(12)$ and the algorithm in class that evaluates a poly of degree $n-1$ on all of the $2n$th root of unity by reducing this to evaluating a poly of degree $(n/2) - 1$ on all of the nth root of unity.

   (b) Do the above two steps for $p(x) = 3x^3 + 7x^2 + x + 8$.

4. (30 points) Prove that, for all $n \geq 0$, both $\sin \frac{2\pi}{2^n}$ and $\cos \frac{2\pi}{2^n}$, can be written using $+, -, \times, /$ and squareroots on integers. (It may be MANY squareroots like $\sqrt{2 + \sqrt{3}}$.) (HINT: You will need some Trigonometric identities and induction.)