Fall 2006 **CMSC 451: Homework 8** William Gasarch

Due Wedensday Nov 15, at Beginning of class

IMPORTANT NOTE: Be CLEAR, Be SHORT. You may lose points for a correct answer if the TA does not understand it.

1. (0 points). Write your name clearly. Staple your HW. READ Chap on Dynamic programming AND your classnotes- the material in All-pairs-shortest-path are NOT, and on NIM games, are not in the book. Also, when and where are the final?

2. (50 points) We define the cost of a path as being the MAX of the weights of the edges on it (instead of being the sum).

   (a) Let $d_1(i,j,k)$ be the cost of going from i to j where we are allowed to use $\{1,2,\ldots,k\}$ as intermediary points. What is $d(i,j,0)$? Write a recurrence for $d(i,j,k)$. Note that when you compute $d(i,j,k)$ you can assume you know $d(i',j',k')$ for all i',j' and for all $k' < k$.

   (b) Let $d_2(i,j,k)$ be the cost of going from i to j where we are allowed to use $\le k$ points as intermediary points. (There is NO RESTRICTION on what the points can be, just on how many there are.) What is $d(i,j,0)$? Write a recurrence for $d(i,j,k)$. Note that when you compute $d(i,j,k)$ you can assume you know $d(i',j',k')$ for all i',j' and for all $k' < k$.

   (c) Devise an algorithm that solves THIS version of All-Pairs-Shortest-path problem in time $O(n^a)$ where $a < 3$.

3. (50 points) Let A be a DFA. Let $R(i,j,k)$ be a regular expression that expresses all words w such that if you start in state i and read in w you end up at state j BUT use AT MOST k intermediary states. (NOTE- this is DIFFERENT from the $R(i,j,k)$ in class.)

   (a) What is $R(i,j,0)$?

   (b) Give a recurrence for $R(i,j,k)$. Note that when you compute $R(i,j,k)$ you can assume you know $R(i',j',k')$ for all i',j' and for all $k' < k$.